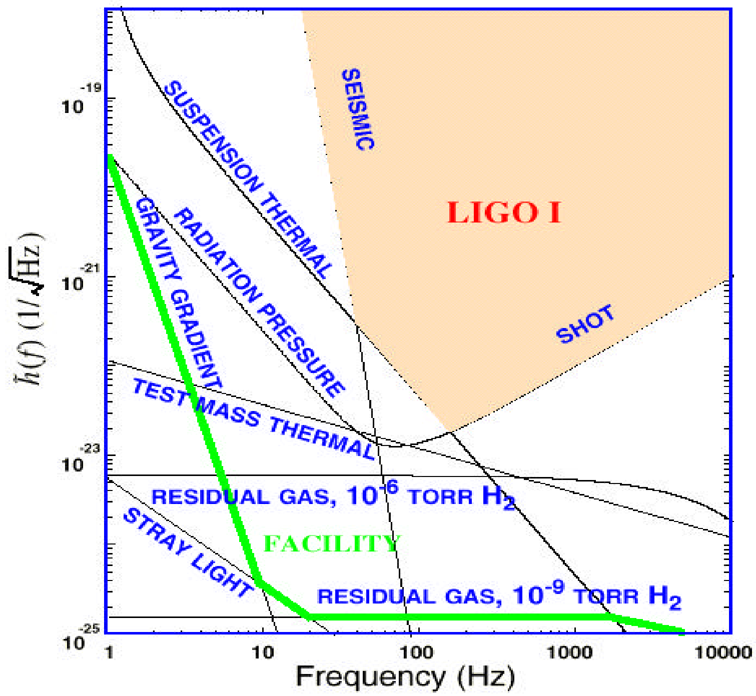
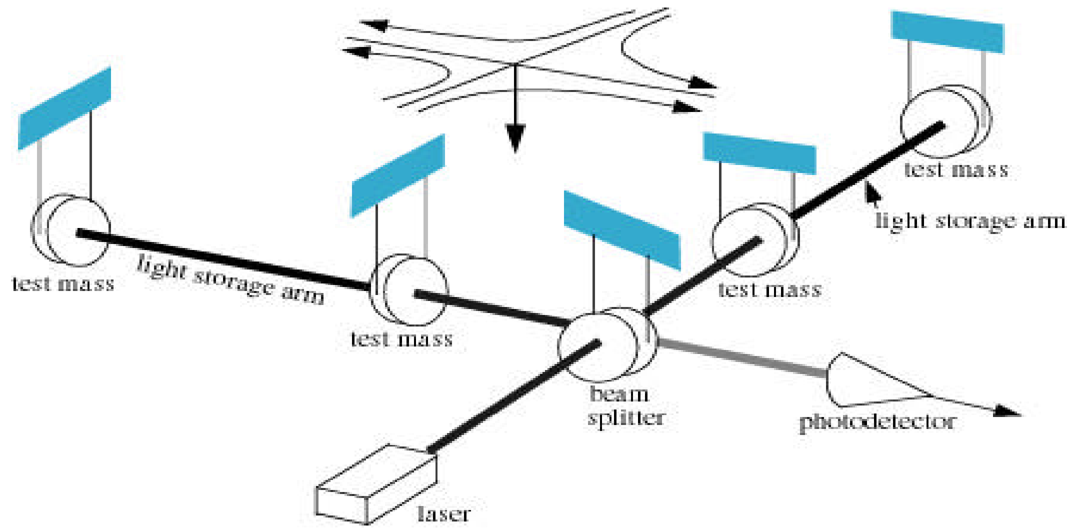
Overview of LIGO core technologies

- IFO phase sensing
- Light storage
- Optical cavities
- Cavity control
- Test mass suspensions
- LIGO control systems
- Noise sources and LIGO subsystems
- Advanced LIGO



Interferometer for GWs

- The concept is to compare the time it takes light to travel in two orthogonal directions transverse to the gravitational waves.
- The gravitational wave causes the time difference to vary by stretching one arm and compressing the other.
- The interference pattern is measured (or the fringe is split) to one part in 10^{10}, in order to obtain the required sensitivity.

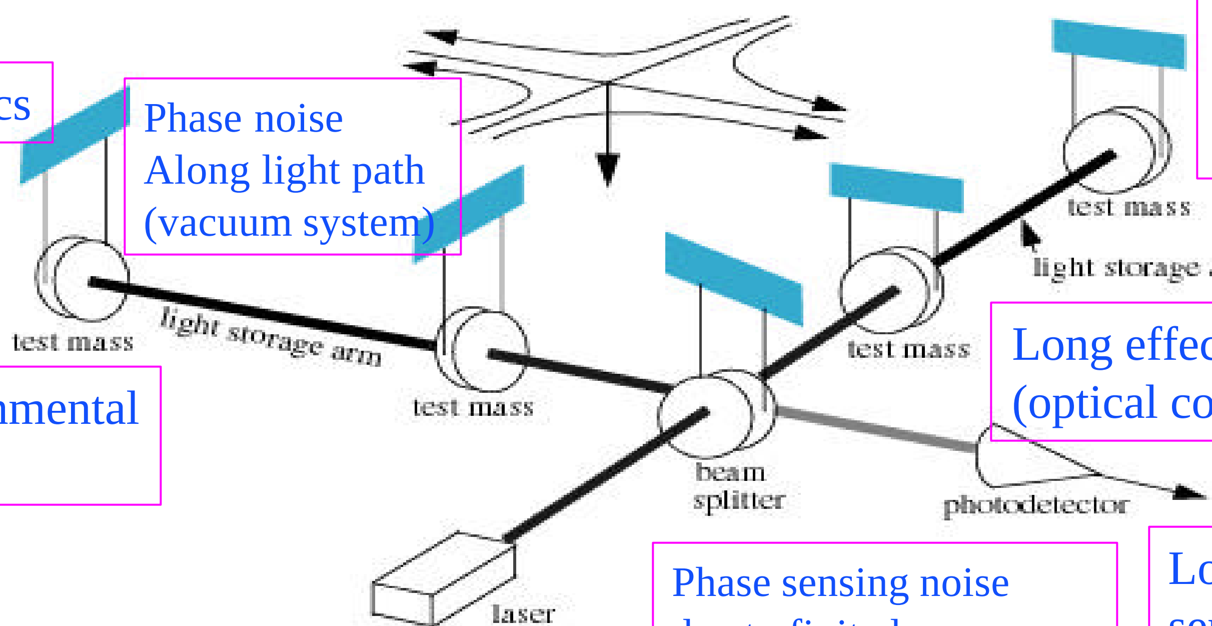


Controlling noise in GW IFOs

Suspended optics

Phase noise
Along light path
(vacuum system)

Test mass
thermal noise
control



Seismic/environmental
noise isolation

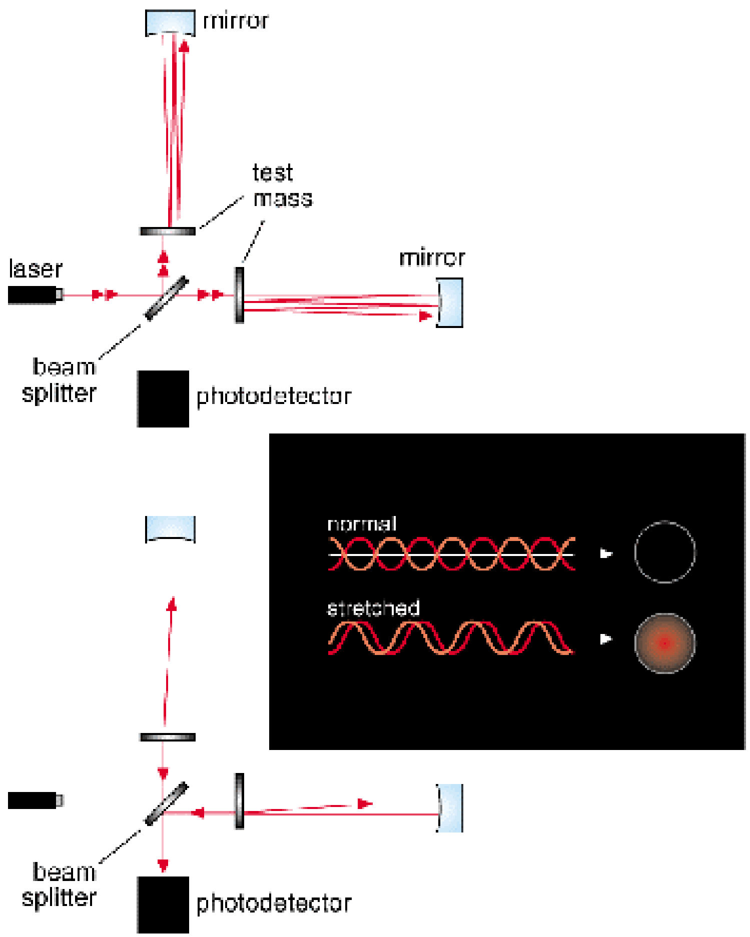
Long effective arm length
(optical configuration)

Phase sensing noise
due to finite laser
power (power recycling)

Low-noise
sensing
electro-optics
and electronics

Extremely stable laser:
- frequency/phase fluctuations
- intensity fluctuations
- Transverse profile

Interferometric phase difference



The effects of gravitational waves appear as a deviation in the phase differences between two orthogonal light paths of an interferometer.

For expected signal strengths, The effect is *tiny*:

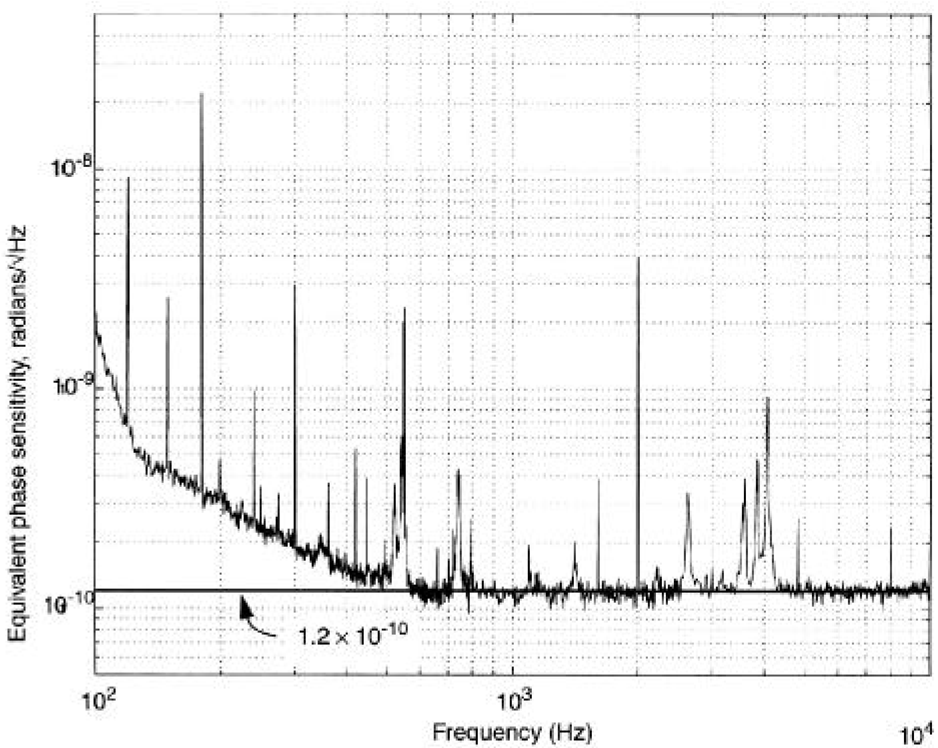
Phase shift of ~10^{-10} radians

The longer the light path, the larger the phase shift…

Make the light path as long as possible!

Phase Noise

splitting the fringe

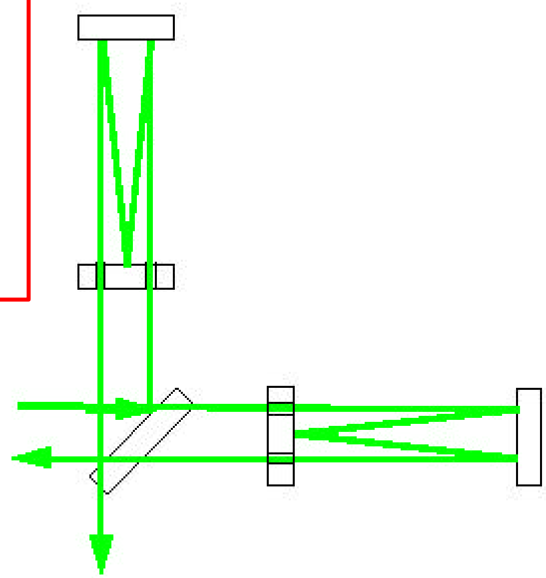


- spectral sensitivity of MIT phase noise interferometer
- above 500 Hz shot noise limited near LIGO I goal
- additional features are from 60 Hz powerline harmonics, wire resonances (600 Hz), mount resonances, etc

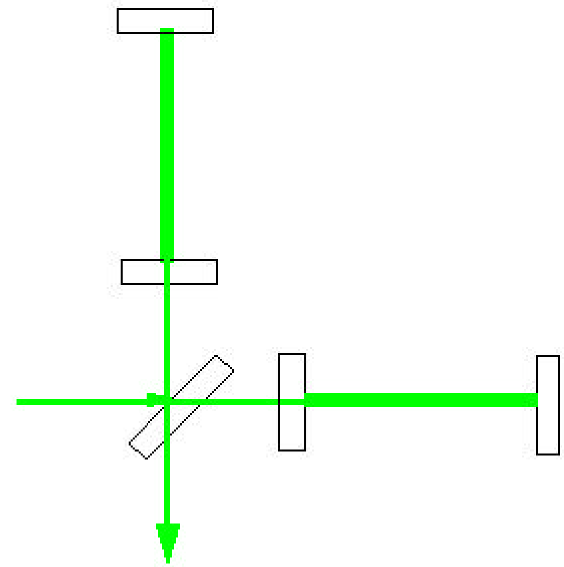
Light storage: folding the arms

How to get long light paths without making *huge* detectors:

Fold the light path!



Delay line interferometer



Fabry Perot interferometer

Simple, but requires large mirrors; limited t_{stor}

(LIGO design) $t_{stor} \sim 3\ msec$

More compact, but harder to control

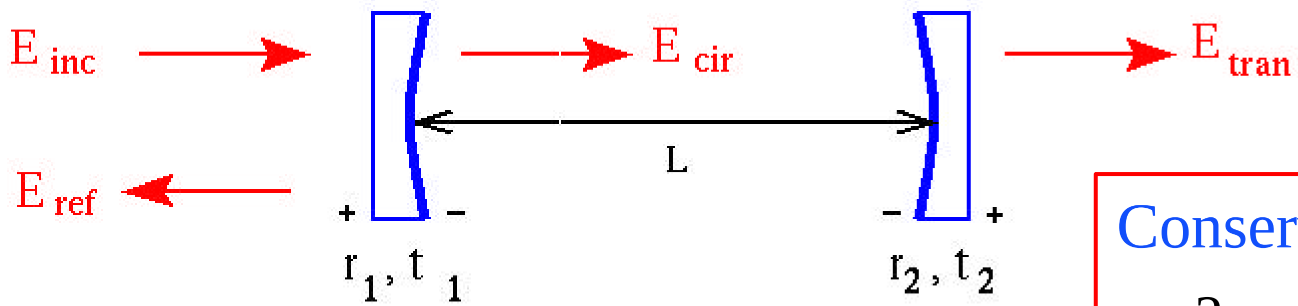
A practical interferometer

- The earth is too noisy at low frequencies…
- Using a pendulum of length l = 50 cm, f_0 ~ 1 Hz, so mass is "free" above ~ 100 Hz

$$f_0 = \frac{1}{2p}\sqrt{\frac{g}{l}} \approx 0.7\ \mathrm{Hz}$$

- A GW with $f_g \sim 100$ Hz $\Rightarrow \lambda_g \sim 3000$ km produces a tiny strain $h = DL / L$
- *We measure* $D f = 4p\, DL/l_{laser} = 4p\, L\, h / l_{laser}$ so to measure small h, need large L
- *But not too large!* If $L > \lambda_g/4$, GW changes sign while laser light is still in arms, cancelling effect on $D f$
- Optimal: $L > \lambda_g/4 \sim 750$ km. But not very practical!
- For more practical length (L ~ 4 km), increase phase sensitivity: $D f = 4p\, DL/l_{laser} \Rightarrow D f = N(4p\, DL/l_{laser})$, *with* $N \sim 200$
- N : Increase number of times light beam hits mirror, so that the light is phase-shifted N times the single-pass length diff DL

Fabry-Perot Cavities



$$E_{cir} = t_1 E_{inc} + r_1 r_2 e^{-2ikL} E_{cir} = \frac{t_1}{1 - r_1 r_2 e^{-2ikL}} E_{inc}$$

$$E_{ref} = r_1 E_{inc} - t_1 r_2 e^{-2ikL} E_{cir} = \frac{r_1 - r_2(1 - L_1) e^{-2ikL}}{1 - r_1 r_2 e^{-2ikL}} E_{inc}$$

$$E_{tran} = t_2 \; e^{-ikL} E_{cir} = \frac{t_1 t_2 e^{-ikL}}{1 - r_1 r_2 e^{-2ikL}} E_{inc}$$

Conservation of energy:

$$r_i^2 + t_i^2 + L_i = 1$$

$$R_i + T_i + L_i = 1$$

When $2kL = n(2p)$, (ie, $L = n\,l/2$),

E_{cir}, E_{tran} maximized $\Rightarrow$ resonance!

And near resonance…

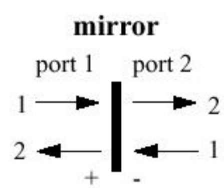
$$E_{ref} = \frac{r_1 - r_2(1-L_1)e^{-2ikL}}{1-r_1 r_2 e^{-2ikL}} E_{inc}$$

$$\boxed{\frac{E_{ref}}{E_{inc}} = \left(\frac{E_{ref}}{E_{inc}}\right)_{res}\left(1-\frac{r_1 r_2}{1-r_1 r_2}2ikdL\right)}$$

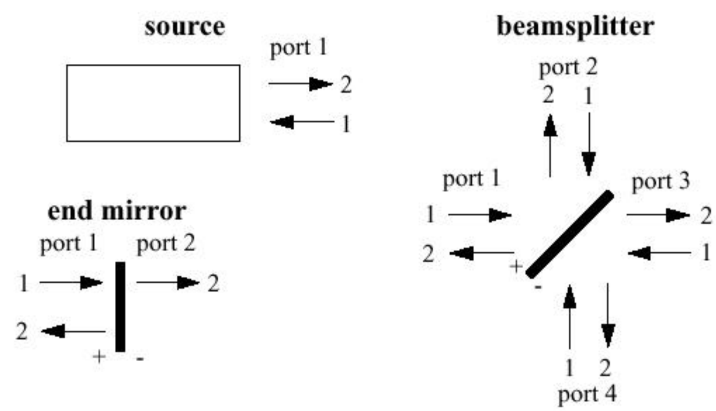
- δL is a tiny quantity; expand $e^{2ik(L+dL)}$ » $(1+2ikdL)$
- Amplitude of reflected field is phase shifted (note the *i*)
- But intensity $|E_{ref}|^2$ is mostly unchanged
- Must detect the phase shift
- Effect can be tremendously amplified by $1/(1-r_1 r_2)$ (bounce number)
- The response is reduced when *dL* varies sinusoidally with frequency:
  - $e^{i4pc(f+df)L}$ » $(1+i\ 4p\ cL\ d\ f)$
  - $f > f_{pole}$ » $(c/4\pi L)\ (1-r_1 r_2)$
- At higher frequencies, IFO response to δL falls off like 1/f (cavity pole)

Field equations, dynamics

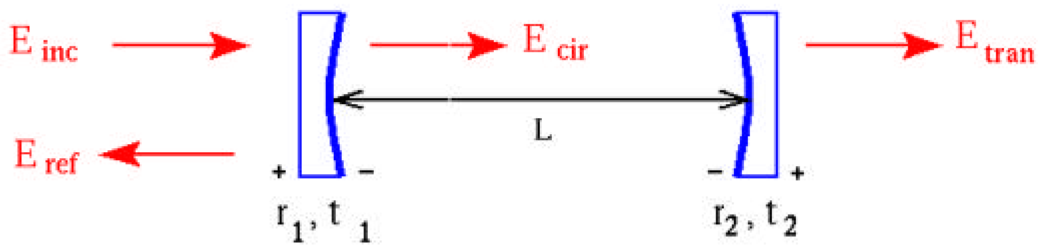
- Any arbitrary configuration of mirrors, beam splitters, sources, defines a set of static fields, and a set of linear relations between them (which depend on phase advances, reflectivities and transmissivities, etc)
- It is thus easy to solve for all the static fields in any configuration
- Dynamics: shake a mirror (or wiggle a source field) at frequency f, and all the fields respond with a wiggle at that frequency.
- Can then calculate the (complex) transfer function between any mirror and any field
- M. Regehr, ***Twiddle***



$$T(\tilde{x}_{mirr}(f) \rightarrow \tilde{E}_{port}(f))$$



Cavity coupling



$$E_{ref} = \frac{r_1 - r_2(1-L)e^{-2ikL}}{1 - r_1 r_2 e^{-2ikL}} E_{inc}$$

- if $r_1 = r_2(1\text{-}L)$, $E_{ref} = 0$ on resonance; optimal coupling
- if $r_1 > r_2(1\text{-}L)$, $E_{ref} > 0$ on resonance; under-coupling
- if $r_1 < r_2(1\text{-}L)$, $E_{ref} < 0$ on resonance; over-coupling

Free Spectral Range: $f_{FSR} = c/2L$

(eg, for 4 km arms, f_{FSR}=37.5 kHz)

LIGO: carrier is resonant in arms, sidebands not; f_{SB} far from f_{FSR}

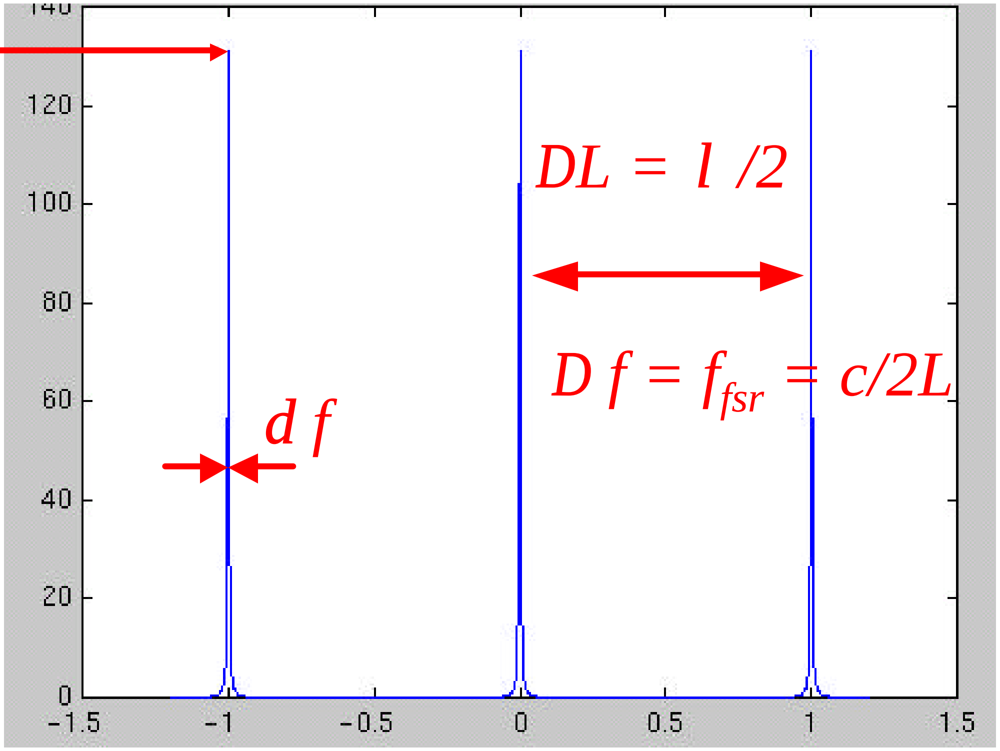
More cavity parameters

- Finesse: peak separation / full width of peak
- Finesse = $F = \frac{p\sqrt{r_1 r_2}}{1 - r_1 r_2}$ = 208 for LIGO 4km arms
- Light storage time = $t_{stor} = \frac{L}{c}\frac{\sqrt{r_1 r_2}}{(1 - r_1 r_2)}$ = 870 µsec for LIGO arms
- Cavity pole = $f_{pole} = 1/(4pt_{stor})$ = 91 Hz for LIGO arms
- Cavity gain = $G_{cav} = \left(\frac{t_1}{1 - r_1 r_2}\right)^2$ = 130 for LIGO arms
- Visibility: $V = 1 - P_{min}/P_{max}$, Power in/out of lock
- LIGO 4km arms: $t_1^2 = 0.03$, $r_2^2 \approx 0.99997$

FP circulating field

Power Gain

$$\left|\frac{E_{circ}}{E_{in}}\right|^2$$



$$Finesse = d\ f/f_{fsr}$$

$$Dn = D(2kL)/2p = Df/f_{fsr} = DL/(l\ /2)$$

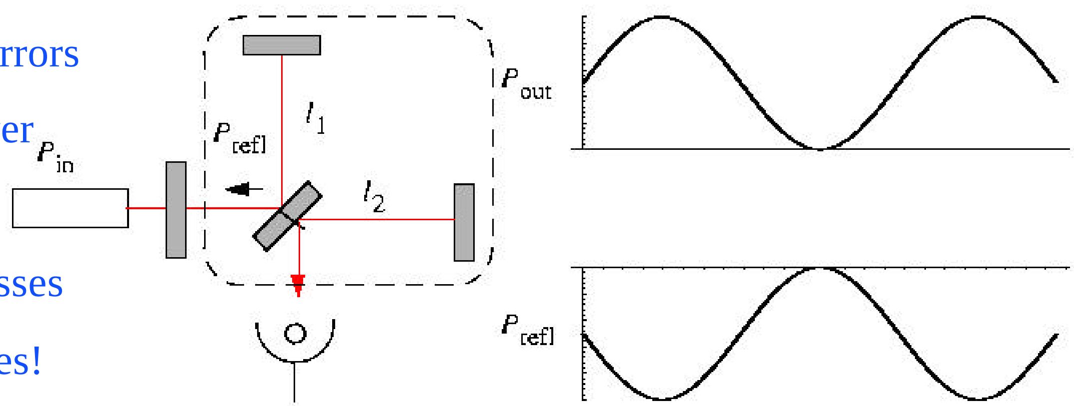
Power recycling

Optimal sensitivity requires high laser power

- predicted sources require shot noise of ~300 W on BS
- suitable lasers produce ~10 W, only ~6W at IFO input

Power Recycling: Make resonant cavity of IFO and *recycling* mirror

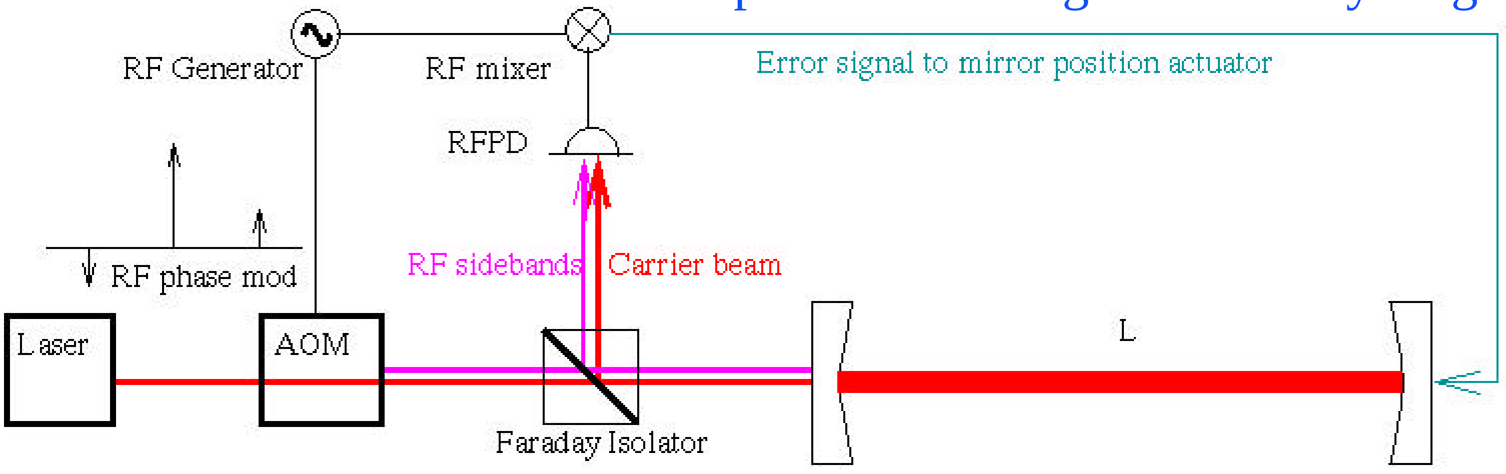
- use IFO at `dark fringe'; then input power reflected back
- known as Recycling of light (Drever, Schilling)
- Gain of ~40 possible, with losses in real mirrors
- allows present lasers to deliver needed power
- increases stored energy
- just extract small amount (or so) if GW passes
- Performance is entirely determined by losses!



Cavity control

Pound-Drever (reflection) locking used to control lengths of all the optical cavities in LIGO

- Phase modulate incoming laser light, producing RF sidebands
- Carrier is resonant in cavity, sidebands are not
- Beats between carrier and sidebands provide error signal for cavity length



Phase modulation of input beam

Phase modulation adds sidebands to the beam:

$$E_{inc} = E_{laser} e^{i(wt + \Gamma \cos \Omega t)} \approx E_{laser} e^{iwt} \left(J_0(\Gamma) + J_{+1}(\Gamma) e^{i\Omega t} + J_{-1}(\Gamma) e^{-i\Omega t} \right)$$

Ω = RF modulation frequency ($\Omega/2\pi \sim 30$ MHz)

Γ = modulation depth

J_i = Bessel functions; $J_{\pm 1} \approx \pm \Gamma/2$ for $\Gamma < 1$

$$E_{ref} = \left(E_0^{ref} + E_{+1}^{ref} e^{i\Omega t} + E_{-1}^{ref} e^{-i\Omega t} \right) e^{iwt}$$

Arrange the length of the cavity, and the value of Ω, so that

•carrier is resonant in FP cavity, sidebands are not,

•so they have different reflection coefficients

•phase of carrier is sensitive to length changes in cavity, sidebands are not

Demodulation

$$S_{ref} = \left(\left|E_0\right|^2 + \left|E_+\right|^2 + \left|E_-\right|^2 \right) + 2\,\mathrm{Re}\left(\left(E_0^* E_+ + E_0 E_-^* \right) e^{i\Omega t} \right) + 2\,\mathrm{Re}\left(E_+^* E_- e^{i2\Omega t} \right)$$

Use an electronic "mixer" to multiply this by

$\cos\Omega t$ or $\sin\Omega t$, average over many RF cycles, to get:

- In-phase demodulated signal $n_I = 2\,\mathrm{Re}\left(E_0^* E_+ + E_0 E_-^* \right)$
- Quad-phase demodulated signal $n_Q = 2\,\mathrm{Im}\left(E_0^* E_+ + E_0 E_-^* \right)$

Which are sensitive to length of cavity (very near resonance)

And can be used as an *error signal* to control cavity length

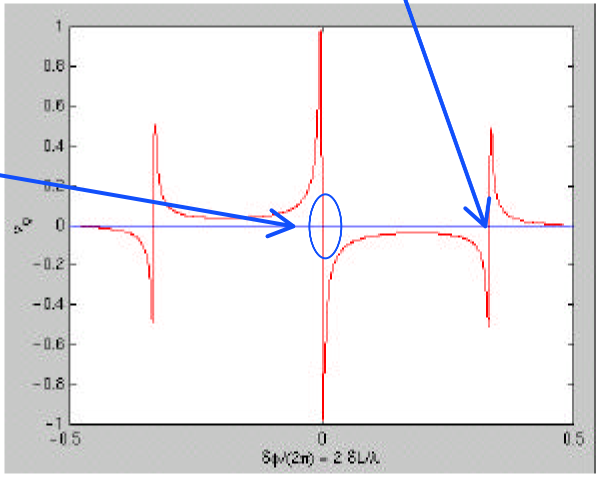
But *only* when it is near resonance!

A mirror will swing wildly until it passes near resonance,

Slow enough for the FB system to "grab" it and hold it there:

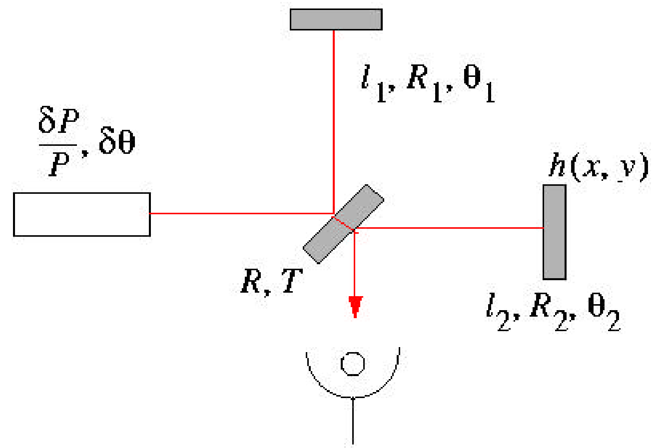
LOCK ACQUISITION

Sideband resonant –
error signal has wrong sign



Schnupp Asymmetry

GW signal (L_-) is measured using light *transmitted* to dark port (**Schnupp locking**, as opposed to reflection locking)



• In absence of GW, dark port is *dark*; carrier power ~ $\sin^2(D f)$, quadratic in $D f = 2kL_-$ for small signal

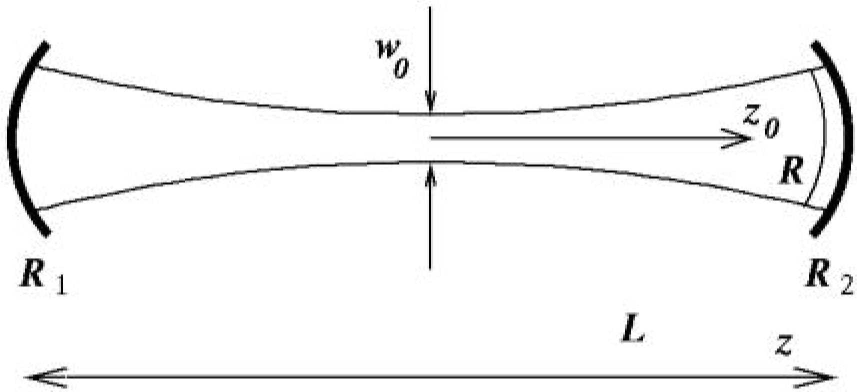
• Add Schnupp (Michelson) asymmetry: l_1 ¹ l_2 ; port is still dark for carrier ($l_1 = l_2 \;\; mod \; l_c$), but sidebands leak out to dark port PD

• Error signal is then *linearly* proportional to amount of carrier light (GW signal)

$$n_Q = 2\,\mathrm{Im}\left(E_0^* E_+ + E_0 E_-^*\right)$$

Transverse profile of beam in FP cavity: Hermite-Gaussian modes

The transverse profile of a beam resonant in a FP cavity is completely determined by L, R_1, R_2, l



Beam waist: $w_0 = \lambda/\pi \; f(L,R_1,R_2)$

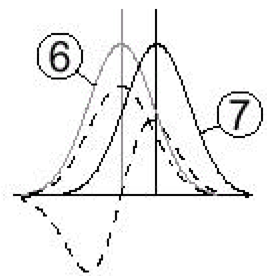
Rayleigh length: $z_0 = \pi w_0^2/\lambda$

- beam waist at position z : $w(z) = w_0\sqrt{1+\left(\frac{z}{z_0}\right)^2}$
- beam ROC at position z : $R(z) = z + \frac{z_0^2}{z}$
- beam *Guoy phase* at position z : $h(z) = \tan^{-1}\left(\frac{z}{z_0}\right)$

Hermite Gaussian Modes

$$E(x,y,z)=\sum a_{mn}U_{mn}(x,y,z), \quad U_{mn}(x,y,z)=U_m(x,z)U_n(y,z)e^{-ikz}$$

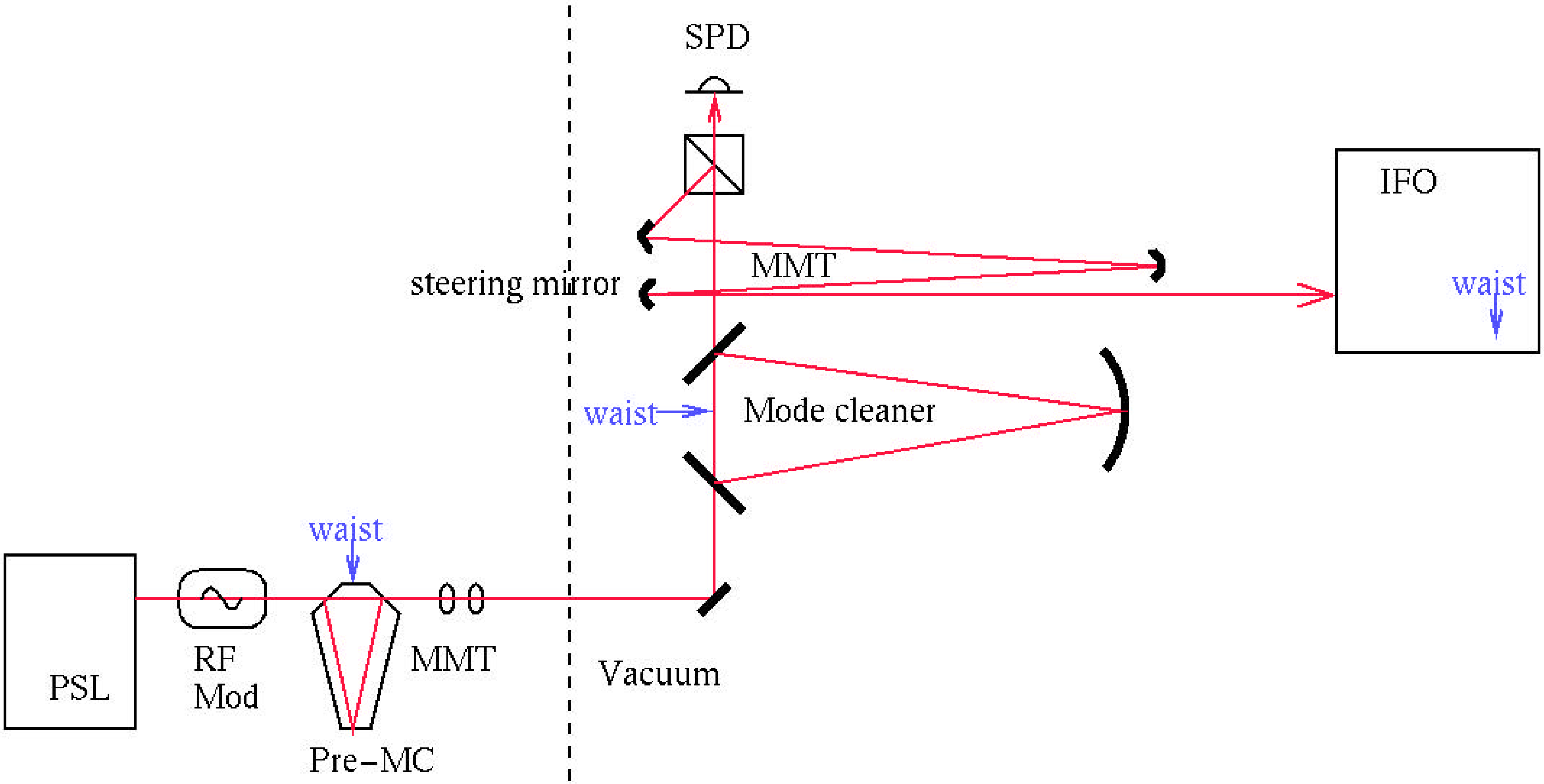
- U_{mn} are *Hermite-Gaussian* or TEM_{mn} transverse modes



$$U_m(x,z)=\sqrt{\frac{\sqrt{2/p}}{2^m m!\,w(z)}}\,H_m\left[\frac{\sqrt{2}x}{w(z)}\right]e^{-x^2\left[\frac{1}{w(z)^2}+\frac{ik}{2R(z)}\right]}e^{i(m+\frac{1}{2})h(z)}$$

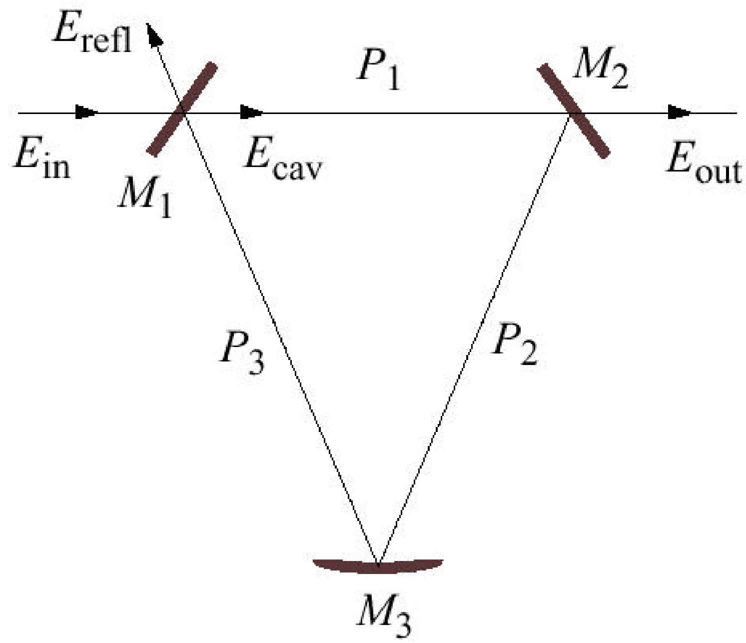
- In a perfect IFO (perfect mirror ROCs, perfect alignment, all cavities *mode matched*), only TEM_{00} mode exists.
- In LIGO cavities, all higher order modes (TEM_{01}, TEM_{10}, etc) represent *beam loss* and *excess noise*;
- Must control mirror imperfections, pitch and yaw, input beam position and direction, mode matching between cavities, *etc*, to minimize this.

Input Optics (IOO)



Mode Cleaner

- Filter out HOMs
- Filter frequency noise from laser
- Triangular MC ensures that reflected light doesn't head back to laser, accessible for reflection locking
- M_3 is very curved, to ensure tight beam (small g-factor)
- Waist is halfway between M_1 and M_2

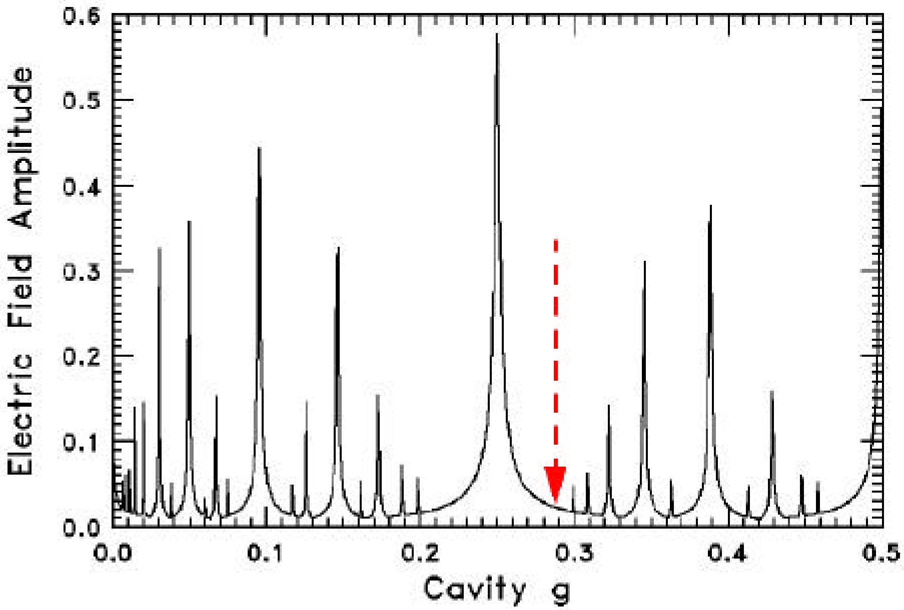


Cavity g-factor

- LIGO Mode Cleaner has two flat and one curved mirror.
- The radius of curvature (ROC) of curved mirror determines g-factor.

$$g = \left(1 - \frac{L}{R}\right)$$

- $g<1$ gives a stable cavity (beam does not diverge as in $R<0, g>1$).
- As g-factor decreases below 1, Guoy phase difference of HOMs gets larger; only one mode resonates in cavity
- g-factor of FP cavity with two curved mirrors is $g = g_1 g_2$, with $g_i=(1-L/R_i)$

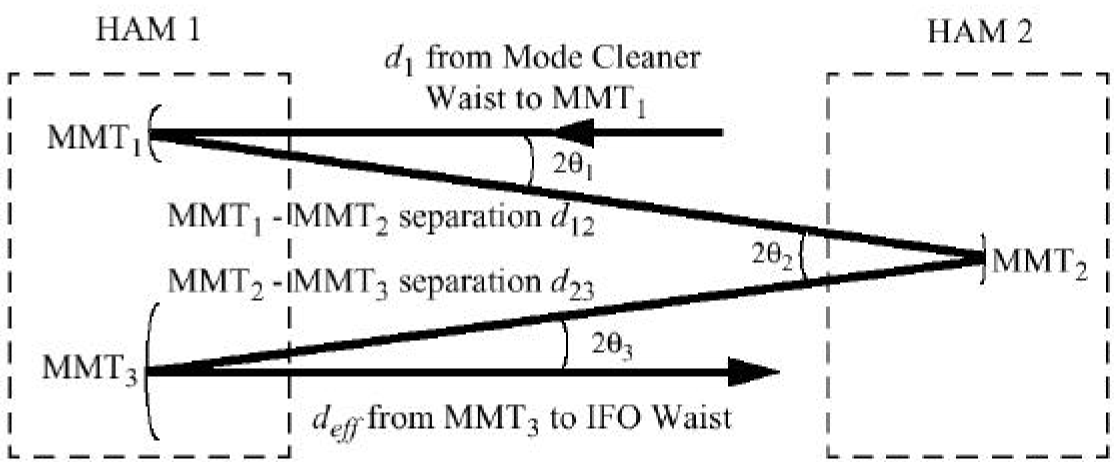


Cavities after cavities within cavities…

- To obtain the laser beam phase stability we need to detect 10^{-10} rad phase shifts, we cascade optical cavities, each longer and more stable than the one before, to quiet the beam frequency fluctuations more and more, over wider and wider frequency band.
- Laser $\rightarrow$ PSL $\rightarrow$ input mode cleaner $\rightarrow$ power recycling cavity $\rightarrow$ arms.
- For advanced LIGO, we'll have a *signal recycling cavity*, and an *output mode cleaner*, as well.
- Each transition requires *mode matching*.

Mode Matching telescope

- Mode Cleaner defines the gaussian beam, with waist in the MC
- The IFO gaussian beam has a waist in the arm cavity
- Need optical telescope to match these beams
- LIGO uses suspended mirrors, rather than transmissive lenses, to minimize noise
- Last MMT mirror steers the beam into IFO

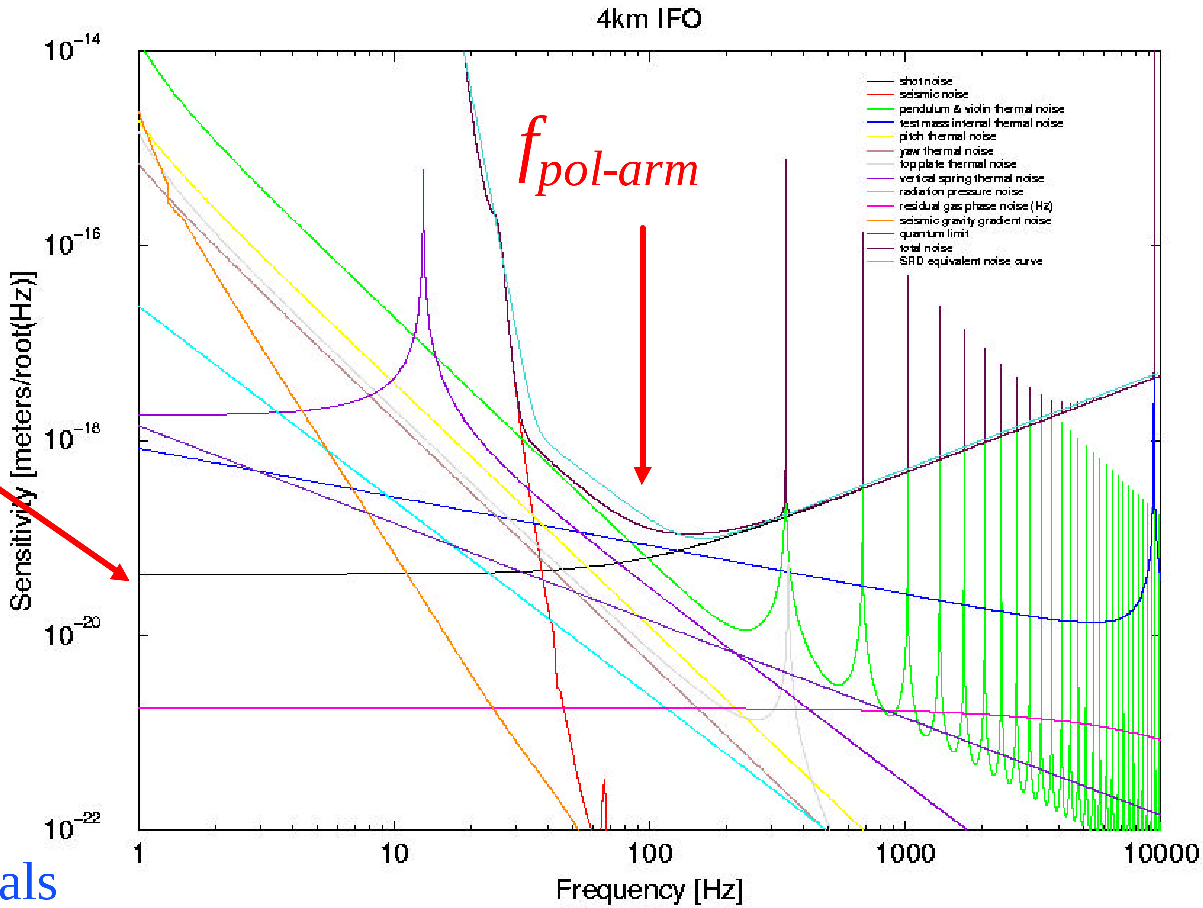


Arm cavity parameters and LIGO sensitivity

As r_{ITM} is increased,
G_{arm} is increased,
$f_{pol\text{-}arm}$ is decreased.

$$h_{dc} \sim 1/\sqrt{G_{arm} P_{laser}}$$

Given other noise sources (seismic, thermal), choose r_{ITM} to optimize Sensitivity to binary inspirals



Contrast

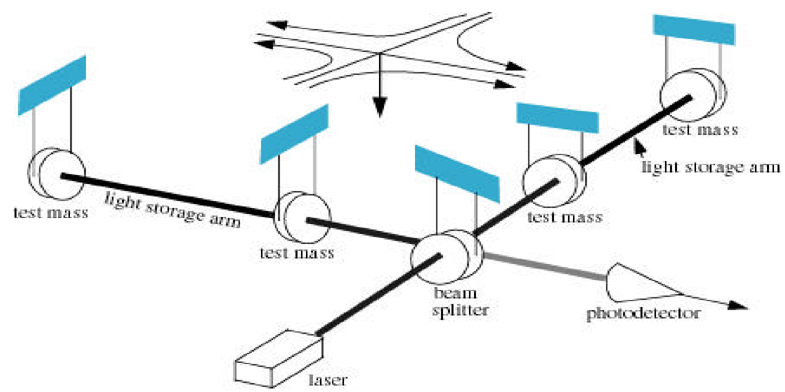
- Contrast is a measure of how perfectly light interferes at beamsplitter

$$C = \frac{P_B - P_D}{P_B + P_D}$$

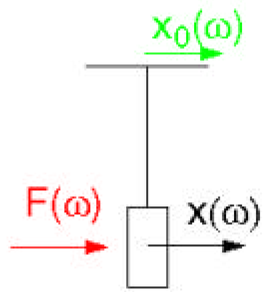
- P_D is minimum carrier power at dark port with both arms in lock
- P_B is maximum carrier power at bright port with both arms out of lock
- Contrast defect 1-C is non-zero due to mode mismatch between arms; imperfect mirrors; etc
- This produces excess noise at GW output, reducing S/N

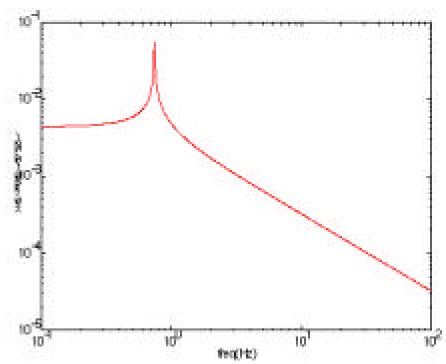
Suspended test masses

- To respond to the GW, test masses must be “free falling”
- On Earth, test masses must be supported against DC gravity field
- The Earth, and the lab, is vibrating like mad at low frequencies (seismic, thermal, acoustic, electrical);
  - can’t simply bolt the masses to the table (as in typical ifo’s in physics labs)
- So, IFO is insensitive to low frequency GW’s
- Test masses are suspended on a pendulum resting on a seismic isolation stack
  - “fixed” against gravity at low frequencies, but
  - “free” to move at frequencies above ~ 100 Hz



“Free” mass:
pendulum at $f \gg f_0$





Pendulum dynamics

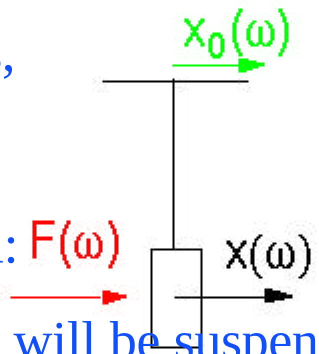
Pendula are wonderful mechanical filters

• They amplify the motion (seismic, thermal, environmental) from suspension point to mass at their resonant frequency f_0

• For frequencies $f >> f_0$, motion is suppressed like f^{-2} (can't yank the bob too fast)

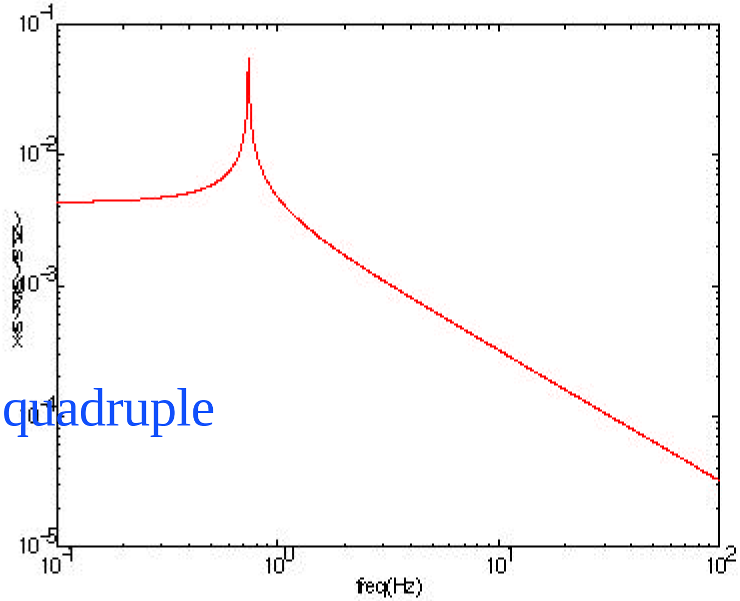
• At such high frequencies,

mass is quiet!



• Pendula can be cascaded:

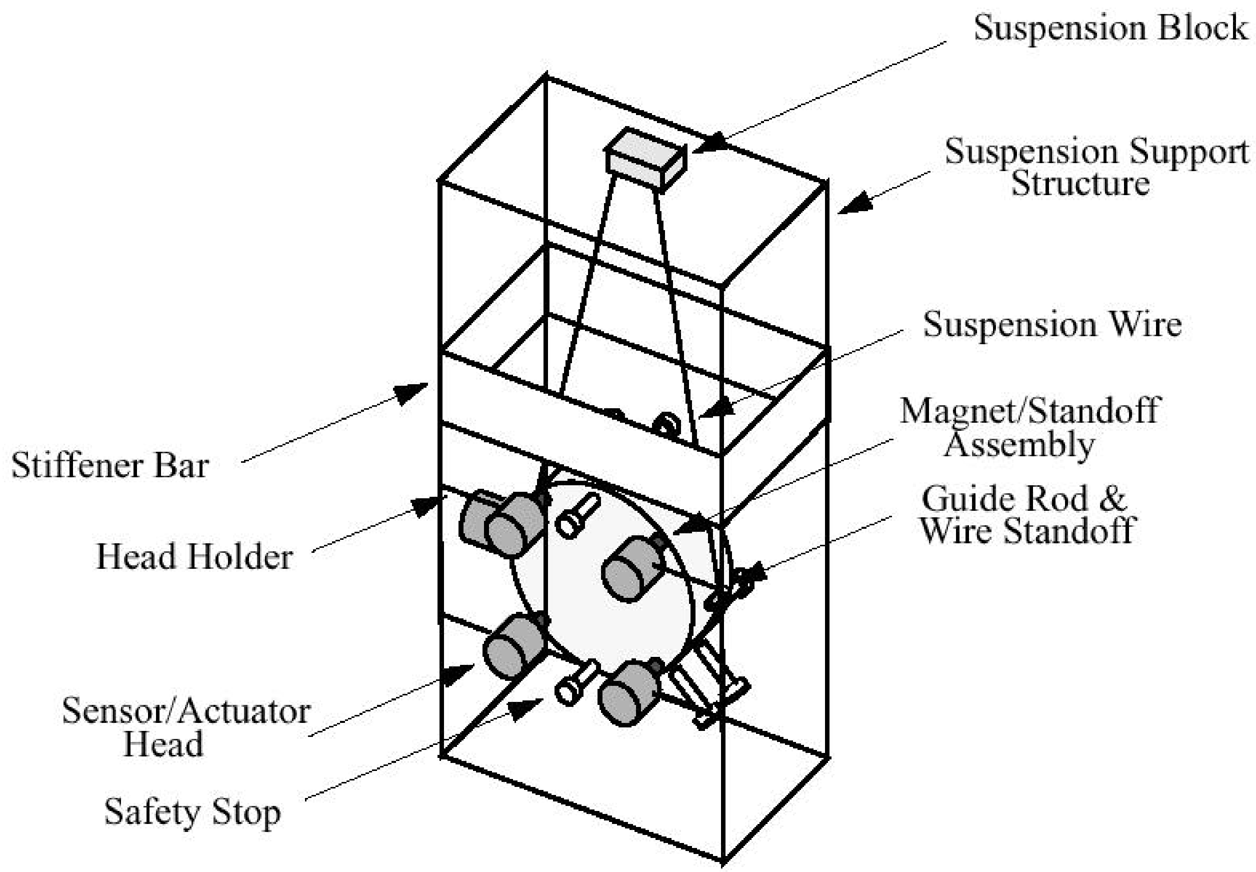
• Advanced LIGO mirrors will be suspended on quadruple pendula



Mirror control

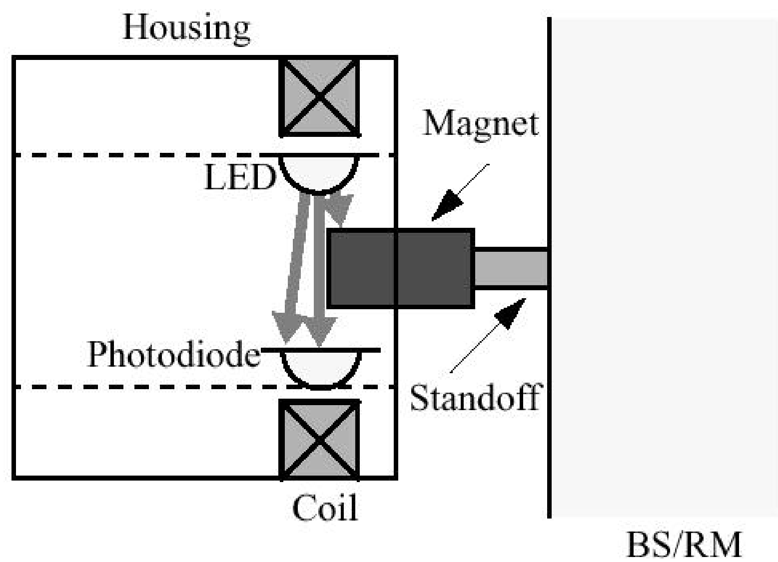
- Seismic isolation system, and pendulum, keep the mirror motion to a minimum.
- Now the mirrors are not being kicked around by the environment (at high frequencies);
- But, being free, they may not be where you need them to be!
- Need active control system to keep mirrors at set points (at/near DC), to keep F-P cavities resonant,
- Without injecting noise at high frequencies
- $\Rightarrow$ Carefully designed feedback servo loops

LIGO I Suspensions



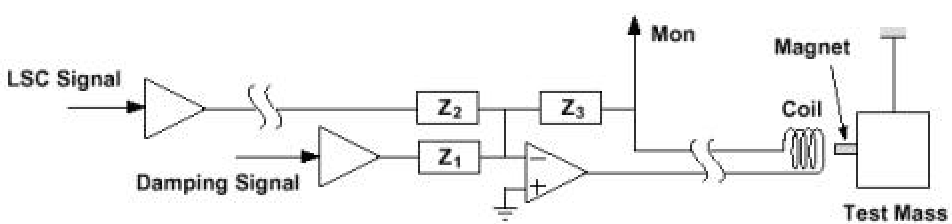
OSEMs

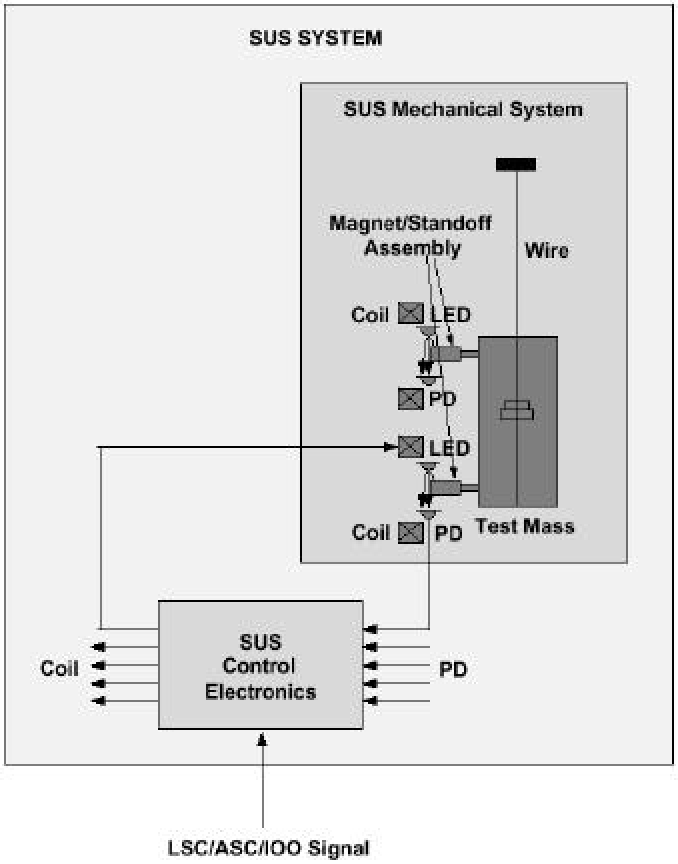
- Five magnets glued to fused Si optic
  - (this ruins the thermal noise properties of the optic – a big problem!)
- LED/PD pair senses position
- Coil pushes/pulls on magnet, against pendulum



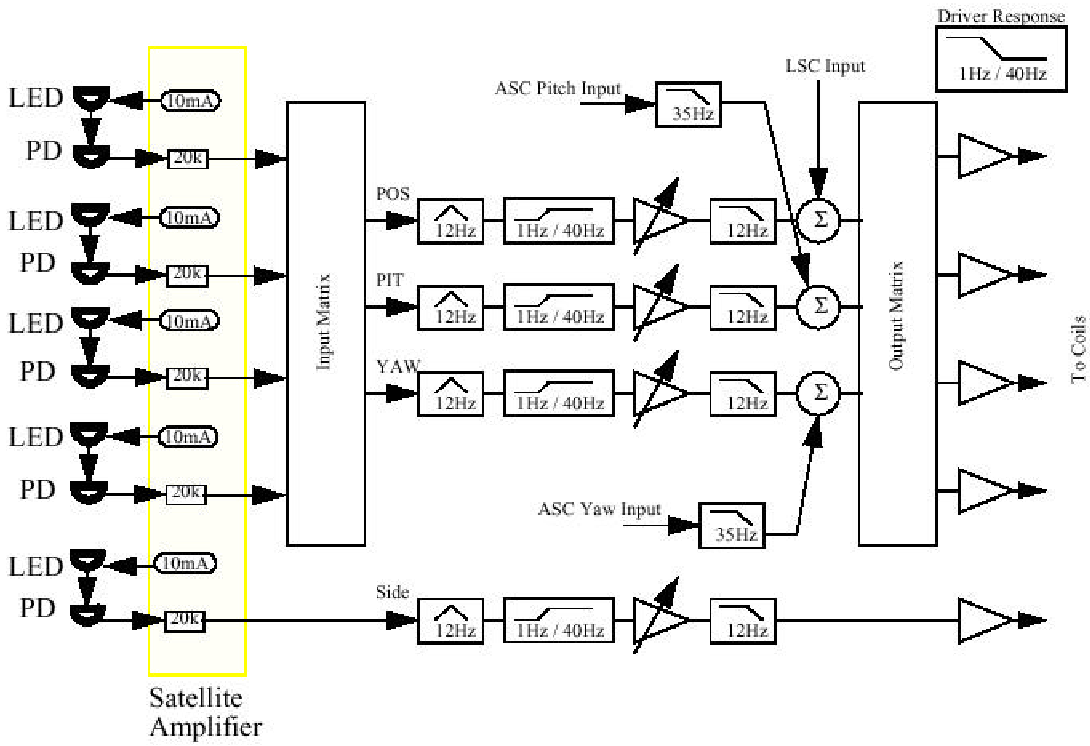
Suspension control system

- Each suspension controller handles one suspension (5 OSEMs)
- Local velocity damping
- Input from LSC and ASC to fix absolute position, pitch, yaw of mirror to keep cavity in resonance

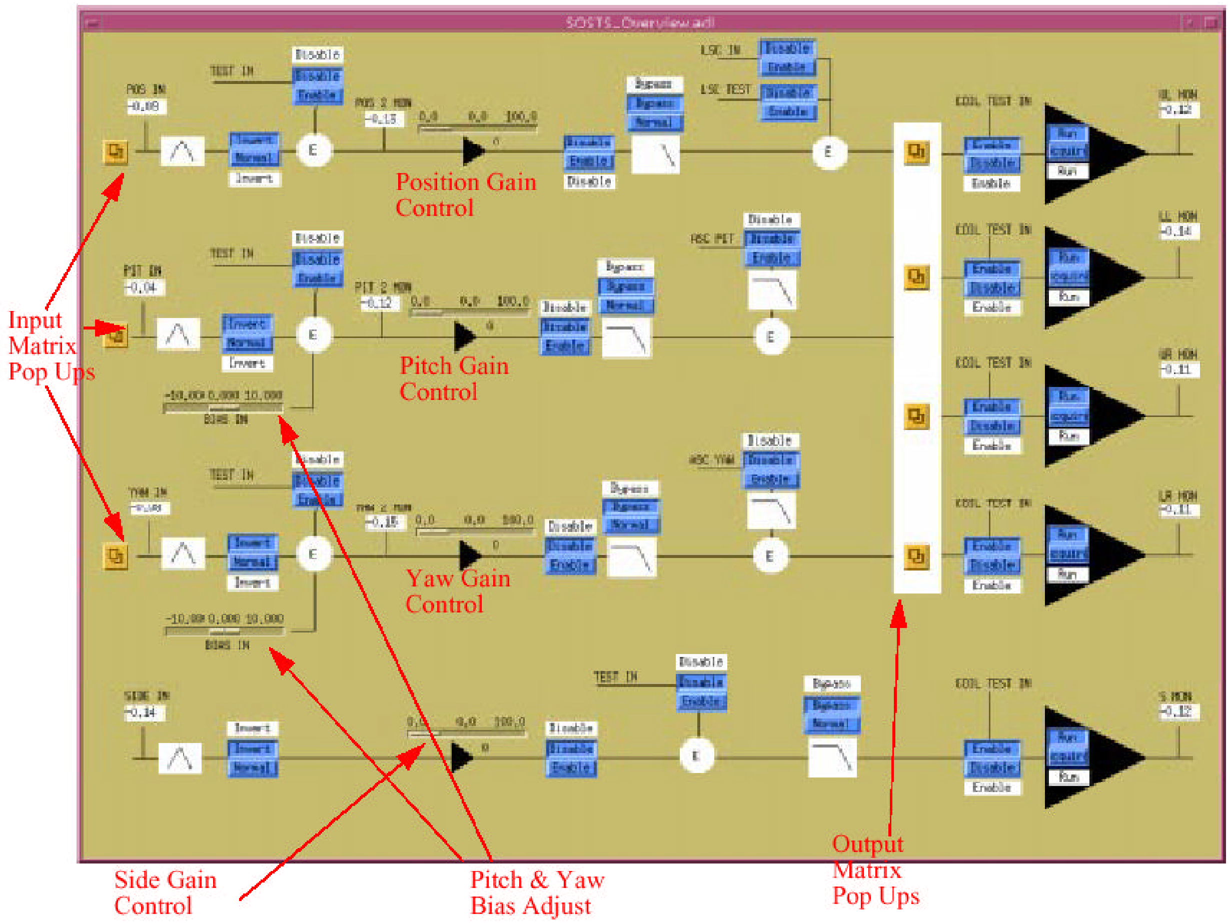




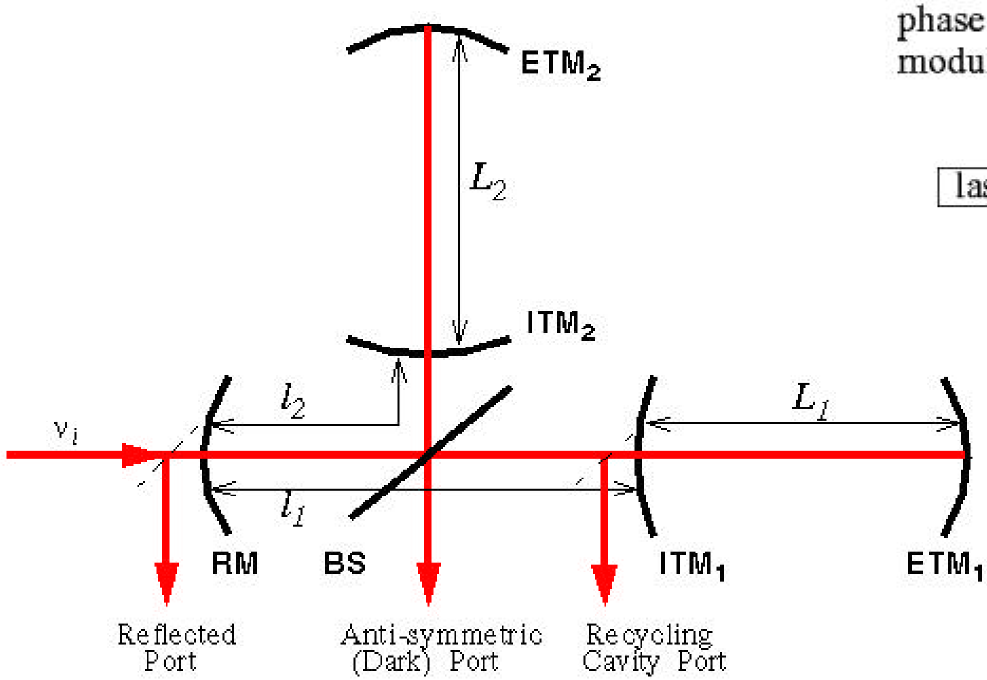
Suspension controller

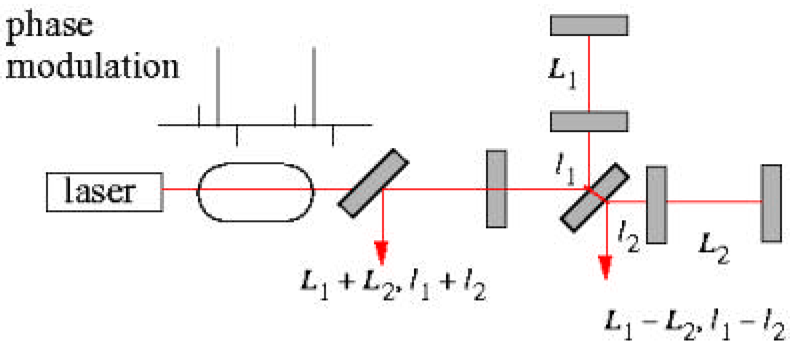


Suspension controller EPICS screen



The control problem in LIGO





- **Four interferometer lengths ⇒ four sensors/actuators**
- **Ten mirror angles ⇒ ten sensors/actuators**