Lecture on signal recycled laser-interferometer gravitational-wave detectors

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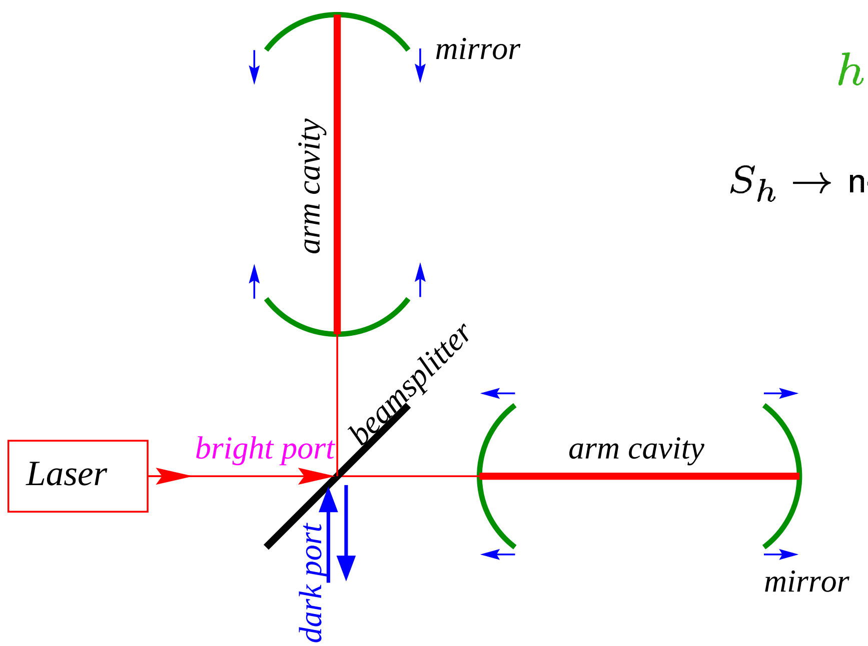
Content

- **Quantum optical noise in conventional interferometers as LIGO-I**
- **Radiation-pressure noise**
- **Free mass standard quantum limit**
- **Quantum optical noise in signal recycled interferometers as LIGO-II**
- **Modified optical-mechanical dynamics**
- **How to improve the sensitivity in the future**

Measurement of very tiny displacements

GEO 600 (Germany-UK), LIGO (USA), TAMA 300 (Japan), Virgo (Italy-France), ...

Frequency band: $10 - 10^4$ Hz



$$h_{\rm rms} = \sqrt{S_h(f)\,\Delta f} = \frac{\Delta L}{L}$$

$S_h \rightarrow$ noise power per unit frequency, $\Delta f \rightarrow$ bandwidth

$L \rightarrow$ arm-cavity length

LIGO-I at $f \sim 100$ Hz: $\Delta L \sim 10^{-16}\,{\rm cm}$

LIGO-II at $f \sim 100$ Hz: $\Delta L \sim 10^{-17}\,{\rm cm} \simeq$ $1/10^8\times$ **radius of hydrogen atom!**

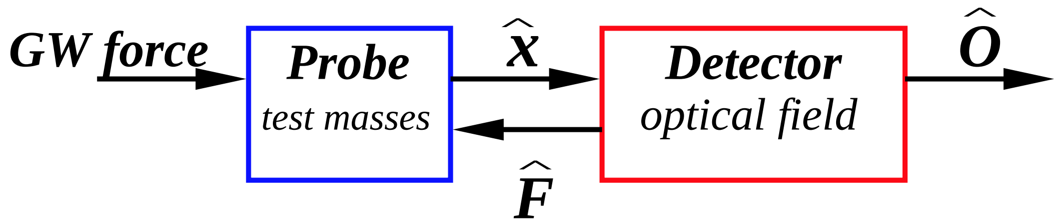
Quantum mechanical formalism to describe optical noise and internal dynamics

Quantum optical noise ...

- Kimble, Levin, Matsko, Thorne & Vyatchanin, [gr-qc/0008026] in Phys. Rev. D
- A.B. & Y.C., [gr-qc/0010011] in Class. Quantum Grav.; [gr-qc/0102012] in Phys. Rev. D; [gr-qc/0107021] in Phys. Rev. D; [gr-qc/0201063] in Class. Quantum Grav.
- Braginsky, Gorodetsky, Khalili, Matsko, Thorne & Vyatchanin, [gr-qc/0109003]

Quantum properties of GW devices

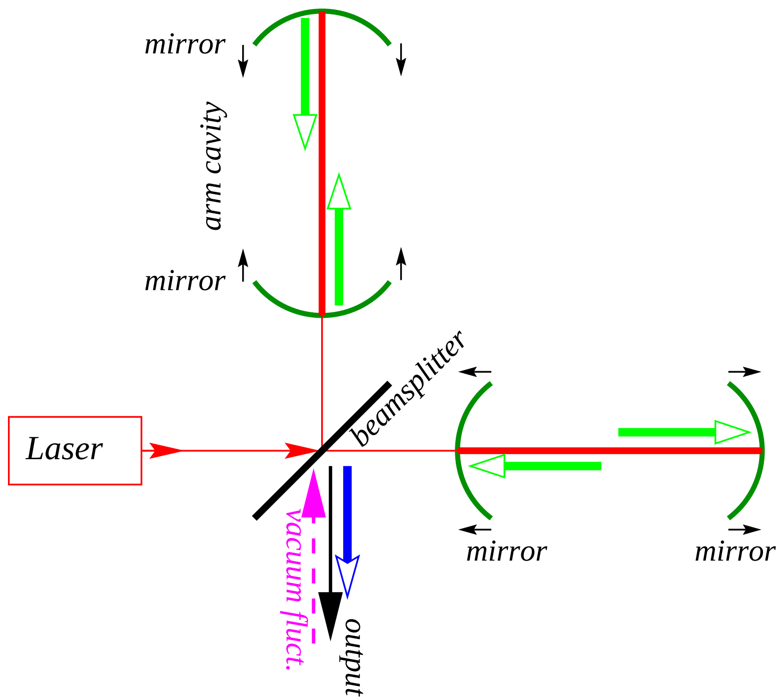
[Braginsky 68-77; Caves, 81; Braginsky & Khalili '92; Braginsky, Gorodetsky, Khalili, Matsko, Thorne & Vyatchanin '01]



How to preclude quantum properties of detector and probe from affecting information we want to extract?

Is there any fundamental quantum limit enforced by probe and detector?

GW interferometers LIGO-I, TAMA, VIRGO, ...



The detector output contains the GW signal, noisy terms scaling such as $\sqrt{I_o}$ (radiation pressure) and $1/\sqrt{I_o}$ (shot noise), and fluctuations associated to initial quantum displacement of mirrors

$I_o \to$ laser light at beamsplitter

Quantum optical-mechanical noise

Noise and signal in output:

$$\Delta h_n(\Omega) = \overbrace{\left[\mathcal{Z}(\Omega) - \frac{1}{\mu\Omega^2}\,\mathcal{F}(\Omega)\right]}^{\text{optical field}} + \overbrace{L\,h(\Omega)}^{\text{GW signal}} + \overbrace{x(\Omega)}^{\text{free test mass}}$$

$\mathcal{Z} \to$ shot noise, $\mathcal{F} \to$ radiation-pressure force, $x \to$ antisym. mode

$\Omega \to$ GW sideband frequency, $\mu = m_{\rm mirror}/4$, $h(\Omega) \to$ GW strain, $L \to$ arm-cavity's length

- GW interferometers: The output noise is not influenced by the test-mass initial quantum state [Braginsky, Gorodetsky, Khalili, Matsko, Thorne & Vyatchanin '01]

$\Rightarrow$ Shot noise and radiation-pressure noise are the only sources of quantum noise

Formalism

Quantum description based on two-photon formalism by Caves & Schumaker '84

$$E(t) = \cos(\omega_o t)\, E_1(t) + \sin(\omega_o t)\, E_2(t), \quad E_{1,2}(t) \propto \int_0^\infty (a_{1,2} e^{-i\Omega t} + a_{1,2}^\dagger e^{i\Omega t}) \frac{d\Omega}{2\pi}$$

$$\omega_o \simeq 10^{15}\,\mathrm{sec}^{-1}, \quad \Omega \simeq 10 - 10^4\,\mathrm{sec}^{-1}, \qquad a_1 \propto a_{\omega_o+\Omega} + a^\dagger_{\omega_o-\Omega} \qquad a_2 \propto a_{\omega_o+\Omega} - a^\dagger_{\omega_o-\Omega}$$

$$[a_1(\Omega), a_2^\dagger(\Omega')] = 2\pi i \delta(\Omega - \Omega'), \qquad [a_1(\Omega), a_1^\dagger(\Omega')] = 0, \qquad [a_2(\Omega), a_2^\dagger(\Omega')] = 0$$

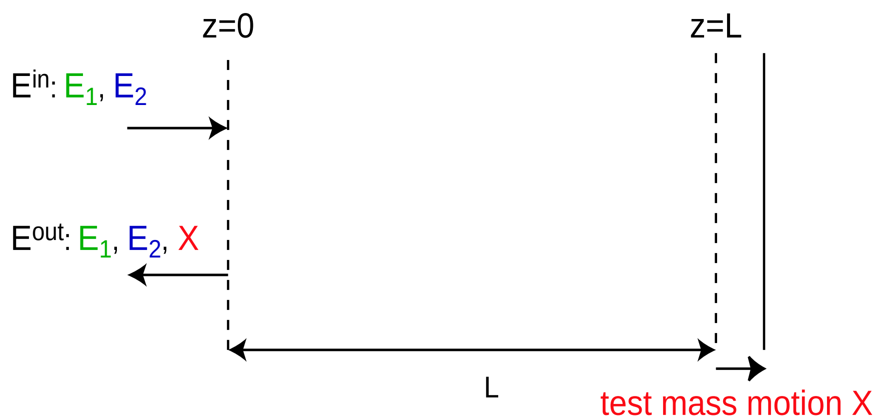
Classically, superimposing monochromatic carrier field $D \cos(\omega_o t)$ (with E_1, $E_2 \ll D$)

$$E_{\rm total}(t) = [D + E_1(t)]\, \cos(\omega_o t) + E_2(t)\, \sin(\omega_o t)$$

$$\approx D \left(1 + \frac{E_1(t)}{D}\right) \cos\left[\omega_o t - \frac{E_2(t)}{D}\right]$$

$E_1(t) \rightarrow$ amplitude modulation (amplitude quadrature) $\qquad E_2(t) \rightarrow$ phase modulation (phase quadrature)

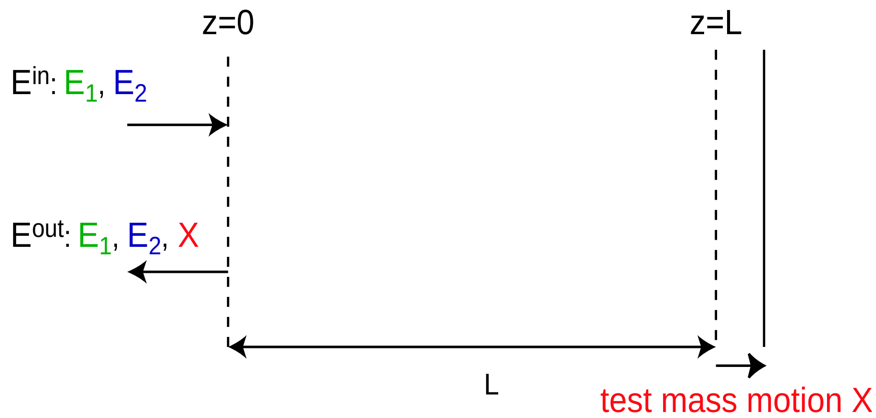
GW sideband generation in single arm cavity



- Input light: $E^{\rm in}(t) = [D + E_1^{\rm vac}(t)]\, \cos(\omega_o t) + E_2^{\rm vac}(t)\, \sin(\omega_o t)$
- Output light [carrier on resonant: $kL = N\pi$ with $N = 0, 1, 2, \ldots, k = \omega_o/c$]

$$\begin{aligned} E^{\rm out}(t) &= E^{\rm in}[t - 2L/c - 2X(t - L/c)/c] \\ &= \left[D + E_1^{\rm vac}(t - 2L/c)\right] \cos(\omega_o t) \\ &+ \left[E_2^{\rm vac}(t - 2L/c) + \underbrace{2kDX(t - L/c)}_{\rm GW\ sideband}\right] \sin(\omega_o t) \end{aligned}$$

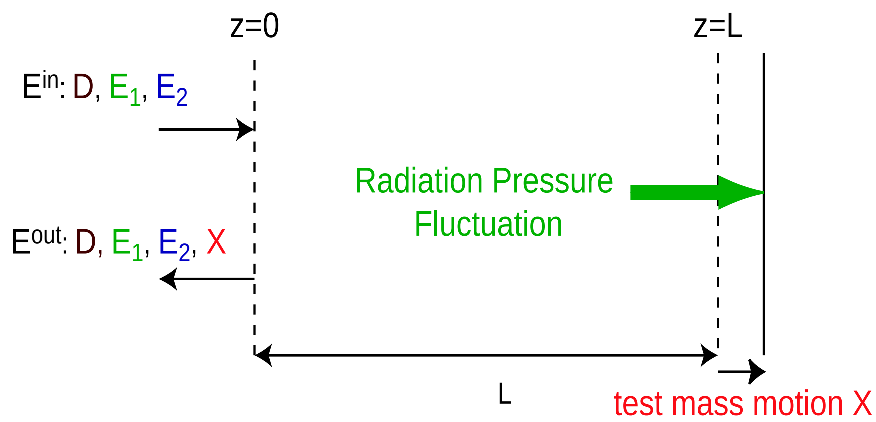
Shot noise in single arm cavity



$$E^{\rm out}(t) \quad \approx \quad D\left(1+\frac{E_1^{\rm vac}(t-2L/c)}{D}\right)\times \cos\left[\omega_o t - \underbrace{\left(\frac{E_2^{\rm vac}(t-2L/c)}{D}+2kX(t-L/c)\right)}_{\text{phase modulation=shot noise+X signal}}\right]$$

- Shot noise $\propto \frac{1}{D} \propto \frac{1}{\sqrt{I_o}}$

Radiation-pressure noise in single arm cavity



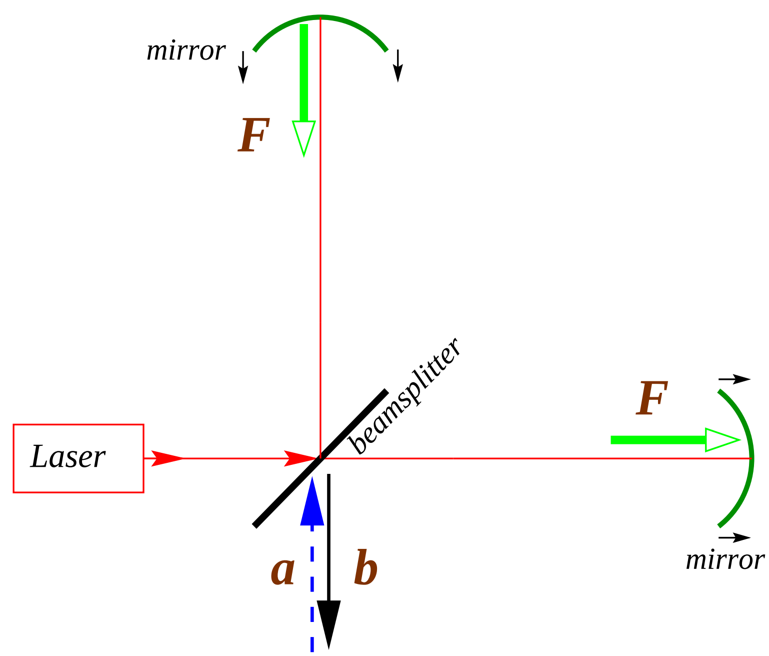
- Carrier amplitude fluctuates $\Rightarrow$ circulating power W in arm cavity fluctuates and hence the radiation-pressure force on the test mass is

$$F_{\rm fluc} = \frac{2W_{\rm fluc}}{c} \propto [E^{\rm in}(t - L/c)]^2 \propto D E_1^{\rm vac}(t - L/c) \qquad \text{[in GW frequency band]}$$

$$\Rightarrow X = \frac{1}{2}Lh + X_{\rm BA} \qquad \ddot{X}_{\rm BA} = F_{\rm fluc}/m \propto D E_1^{\rm vac}(t - L/c)$$

- Radiation pressure noise $\propto D \propto \sqrt{I_o}$

Michelson GW interferometer



Input-output relation of a Michelson GW interferometer

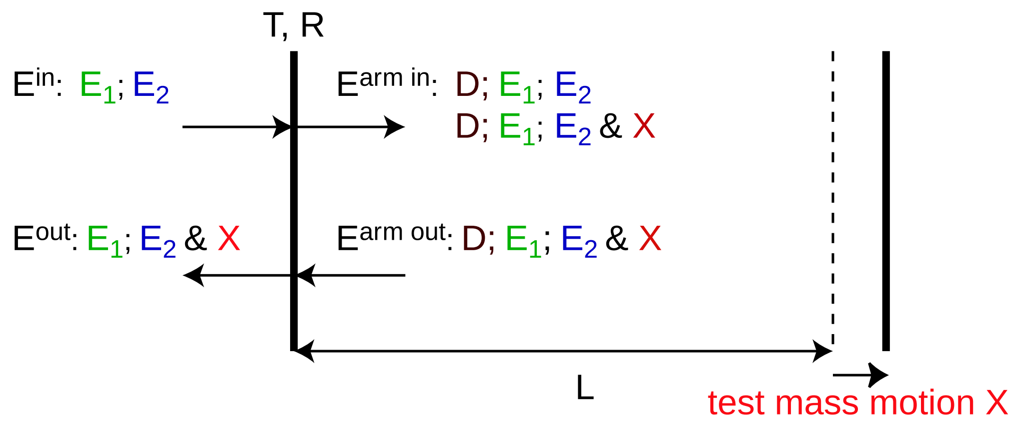
Combining in Fourier domain the previous results

$$\begin{aligned}(\text{amplitude}) \quad b_1(\Omega) &= a_1(\Omega)e^{2i\beta} \\ (\text{phase}) \quad b_2(\Omega) &= \left[a_2(\Omega) - \mathcal{K}a_1(\Omega)\right] e^{2i\beta} + \sqrt{2\mathcal{K}}\frac{h_{\mathrm{GW}}}{h_{\mathrm{SQL}}}e^{i\beta}\end{aligned}$$

$$2\beta = 2\Omega L/c, \qquad \mathcal{K} = \frac{8I_c\omega_o}{m\Omega^2c^2}, \qquad h_{\mathrm{SQL}} = \sqrt{\frac{4\hbar}{m\Omega^2L^2}}$$

Let us consider conventional interferometers but with FP cavities in the arms . . .

Resonant FP cavities in the arms: conventional interferometers



$E^{\rm arm\ out}$ is fed back into the arm, along with vacua from outside, but in simple manner:

$$\text{amplitude modulation} \quad \rightarrow \quad \text{amplitude modulation}$$

$$\text{phase modulation} \quad \rightarrow \quad \text{phase modulation}$$

$\Rightarrow$ Input-output relation has the same form but β and $\mathcal{K}$ change due to cavity-pole effect

FP conventional interferometer: input-output relation

The output can be expressed in terms of the quadrature fields b_1 and b_2 which are function of quantum-vacuum fluctuations a_1 and a_2 entering the interferometer dark port

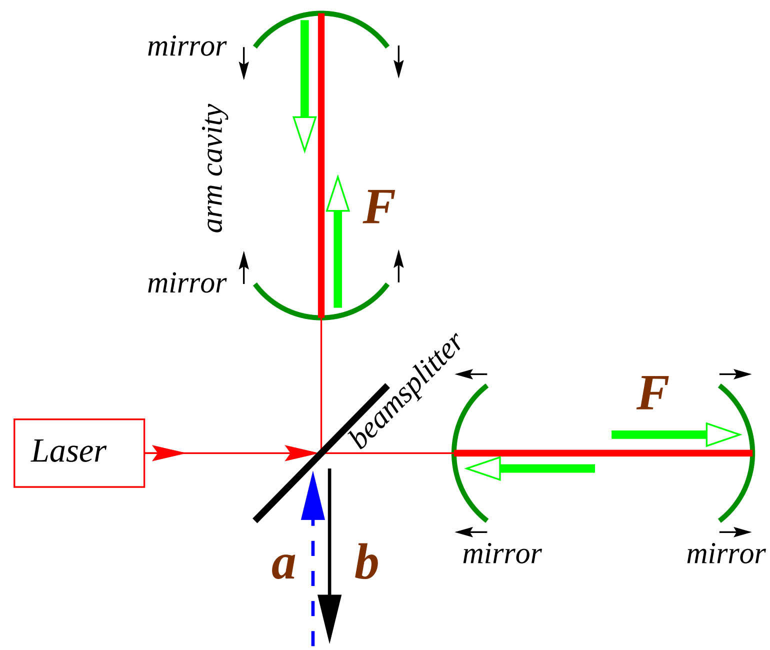
$$b_1(\Omega) = a_1(\Omega)\, e^{2i\beta}$$

$$b_2(\Omega) = [a_2(\Omega) - \mathcal{K}\, a_1(\Omega)]\, e^{2i\beta} + \sqrt{2\mathcal{K}}\, \frac{h_{\rm GW}}{h_{\rm SQL}}\, e^{i\beta}$$

$$\mathcal{K} = \frac{8 I_o \omega_o}{\Omega^2\,(\gamma^2+\Omega^2) m L^2} \qquad h^2_{\rm SQL} = \frac{8\hbar}{m \Omega^2 L^2}$$

$$\gamma = \frac{Tc}{4L} \qquad 2\beta = 2\arctan\frac{\Omega}{\gamma}$$

$$h_n(\Omega) \propto e^{i\beta}\,[\underbrace{\frac{1}{\sqrt{I_o}}\, a_2(\Omega)}_{\text{shot noise}} - \underbrace{\frac{\sqrt{I_o}}{m}\, a_1(\Omega)}_{\text{rad. press.}}] + h_{\rm GW}$$



Conventional interferometer: spectral density

$$h_n(\Omega) \propto e^{i\beta}\,[\underbrace{\frac{1}{\sqrt{I_o}}\,a_2(\Omega)}_{\text{shot noise}} - \underbrace{\frac{\sqrt{I_o}}{m}\,a_1(\Omega)}_{\text{rad. press.}}] + h_{\mathrm{GW}}$$

$$\pi\delta(\Omega-\Omega')\,S_h(\Omega) =< 0_a|h_n(\Omega)\,h_n^\dagger(\Omega')|0_a >$$

$$< 0_a|a_i(\Omega)\,a_j^\dagger(\Omega')|0_a >= \pi\delta(\Omega-\Omega')\,\delta_{ij}$$

If quadrature b_2 (the phase quadrature) is measured:

$$S_h(\Omega) = \frac{h^2_{\mathrm{SQL}}}{2}\,\left(\mathcal{K}+\frac{1}{\mathcal{K}}\right)$$

At high frequency (shot noise): $S_h^{\text{high freq.}}(\Omega) \propto \Omega^2/I_o$

At low frequency (rad.-press.–noise): $S_h^{\text{low freq.}}(\Omega) \propto I_o/\Omega^4$

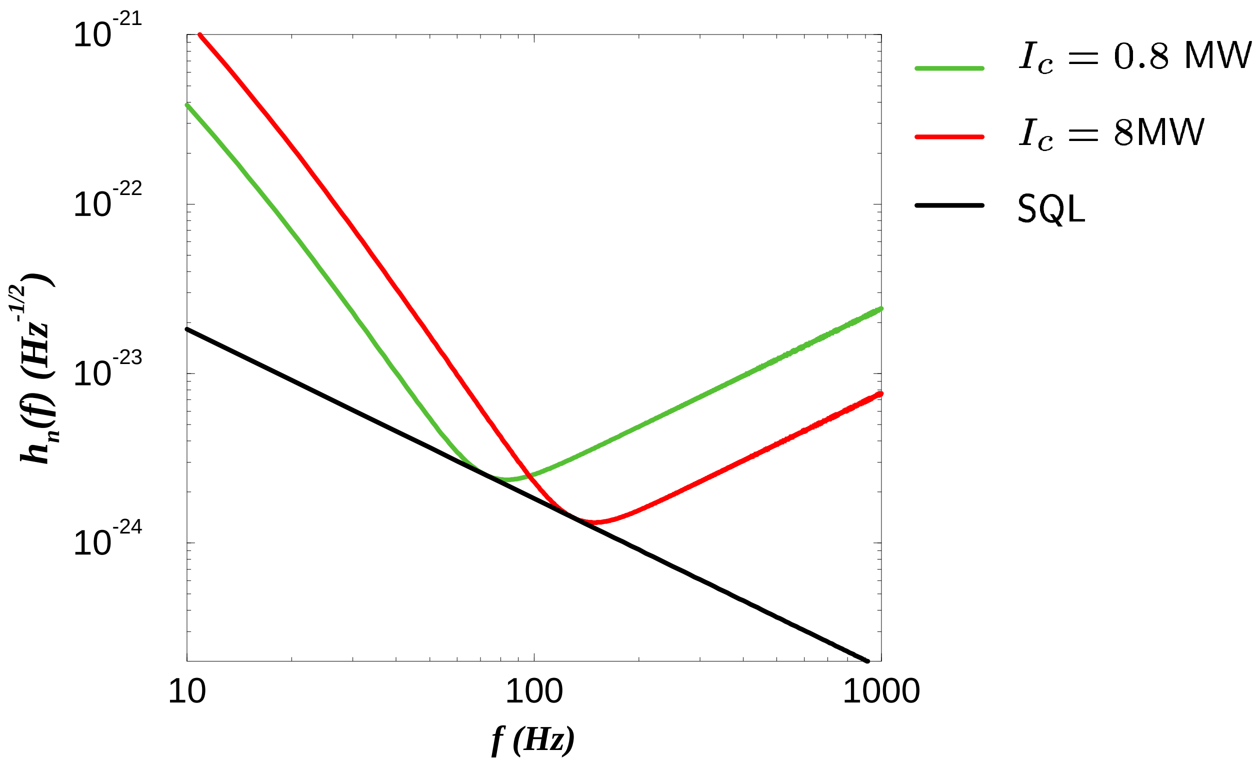
$$\mathcal{K} = \frac{8 I_o \omega_o}{\Omega^2\,(\gamma^2+\Omega^2) m L^2} \qquad h^2_{\mathrm{SQL}} = \frac{8\hbar}{m\Omega^2 L^2} \qquad \gamma = \frac{Tc}{4L} \qquad 2\beta = 2\arctan\frac{\Omega}{\gamma}$$

Optical-noise curves for conventional GW interferometers

$$S_h(\Omega) = \frac{h^2_{\rm SQL}}{2}\left(\mathcal{K} + \frac{1}{\mathcal{K}}\right)$$

$$S_h^{\rm high\,freq.}(\Omega) \propto \Omega^2/I_o$$

$$S_h^{\rm low\,freq.}(\Omega) \propto I_o/\Omega^4$$



- $I_c = 2I_o/T$ → laser light circulating in arm cavity

- $$\sqrt{S_h^{\rm SQL}} \equiv h_{\rm SQL}(\Omega) = \sqrt{\frac{2\hbar}{\mu\,\Omega^2\,L^2}}$$

Free mass SQL for GW interferometers

[Braginsky & Khalili ‘92]

Noise spectral density (noise power per unit frequency)

$$\boxed{S_h(\Omega) = S_h^{\rm shot} + S_h^{\rm rad\ press} + 2\, S_h^{\rm corr}}$$

$$S_h^{\rm shot} \propto S_{\mathcal{Z}\mathcal{Z}} \qquad S_h^{\rm rad\ press} \propto S_{\mathcal{F}\mathcal{F}} \qquad S_h^{\rm corr} \propto S_{\mathcal{Z}\mathcal{F}}$$

$$\boxed{S_h^{\rm shot}\, S_h^{\rm rad\ press} - |S_h^{\rm corr}|^2 \geq S_h^{\rm SQL}/4} \qquad S_h^{\rm SQL}(\Omega) \equiv h^2_{\rm SQL}(\Omega) = \frac{2\hbar}{\mu\,\Omega^2\,L^2}$$

Standard configuration of conventional interferometer (LIGO-I, TAMA, VIRGO)

$$S_h^{\rm corr} = 0 \quad \Rightarrow \quad \boxed{S_h(\Omega) \equiv h_n^2(\Omega) \geq S_h^{\rm SQL}(\Omega)}$$

Similarity with “Heisenberg microscope”

[Braginsky, 68-70; Caves, 81]

Naive derivation of SQL: independent measurements of free test-mass displacements

If positions measured with high precision then test-mass momenta perturbed (Heisenberg uncertainty principle)

As time passes, $x(t) = x_0 + \frac{p_0}{\mu}\,t$, momentum perturbations produce position uncertainties

If momentum perturbations and measurement errors are not correlated

$\Delta x(t)\,\Delta x(t') \geq \frac{\hbar|t-t'|}{2\mu} \quad \Rightarrow \quad$ minimum possible spectral density

$\Rightarrow \quad S_h^{\rm SQL}(\Omega) = \frac{2\hbar}{\mu\,\Omega^2\,L^2}$ for GW signal $h = \frac{\Delta L}{L}$

Building up *static* correlations in conventional interferometers

$$h_n(\Omega) \propto e^{i\beta}\,[\underbrace{\frac{1}{\sqrt{I_o}}\,a_2(\Omega)}_{\text{shot noise}} - \underbrace{\frac{\sqrt{I_o}}{m}\,a_1(\Omega)}_{\text{rad. press.}}] + h_{\rm GW}$$

$$\pi\delta(\Omega-\Omega')\,S_h(\Omega) =< 0_a|h_n(\Omega)\,h_n^\dagger(\Omega')|0_a >$$

$$< 0_a|a_i(\Omega)\,a_j^\dagger(\Omega')|0_a >= \pi\delta(\Omega-\Omega')\,\delta_{ij}$$

If quadrature b_ζ is measured: $b_\zeta(f) = \cos\zeta\,b_1(f) + \sin\zeta\,b_2(f)$ with ζ homodyne angle

$$h_n(\Omega) \propto e^{i\beta}\,\{\underbrace{\frac{1}{\sqrt{I_o}}\,[a_2(\Omega) + \tan\zeta\,a_1(\Omega)]}_{\text{shot noise}} - \underbrace{\frac{\sqrt{I_o}}{m}\,a_1(\Omega)}_{\text{rad. press.}}\}$$

$S_h^\zeta(\Omega) = \frac{h^2_{\rm SQL}}{2}\left[\frac{1}{\mathcal{K}} + \frac{(\tan\zeta-\mathcal{K})^2}{\mathcal{K}}\right]$, by choosing $\tan\zeta = \mathcal{K} \Rightarrow$ only shot noise!

[Matsko, Vyatchanin & Zubova '93,'98]

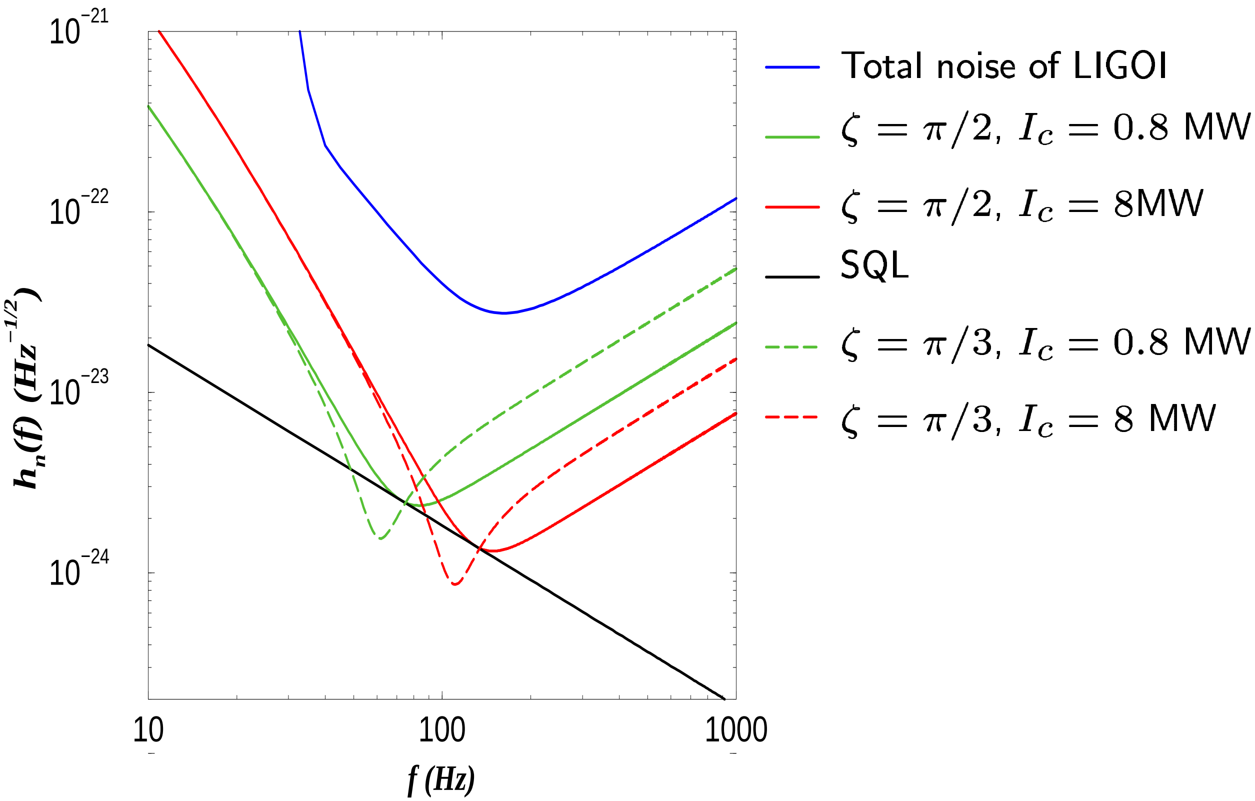
Optical-noise curves for conventional GW interferometers

$$b_\zeta(f) = \cos\zeta\, b_1(f) + \sin\zeta\, b_2(f)$$

$b_i \rightarrow$ e.m. quadrature

- ζ homodyne angle

- $I_c = 2I_o/T \rightarrow$ laser light circulating in arm cavity



Free-mass SQL is beaten if correlations are built up *statically* during read-out process

[Matsko, Vyatchanin & Zubova '93,'98]

Building up correlations by changing the dynamics

- **Optical bar GW detectors: test mass behaves effectively as an oscillator . . .**

  [Braginsky, Gorodetsky & Khalili '97; Braginsky & Khalili '99]

- **Signal-recycling interferometers** (GEO, LIGO-II)

  [Drever '82; Vinet et al '88; Meers '88; Mizuno et al. '93]

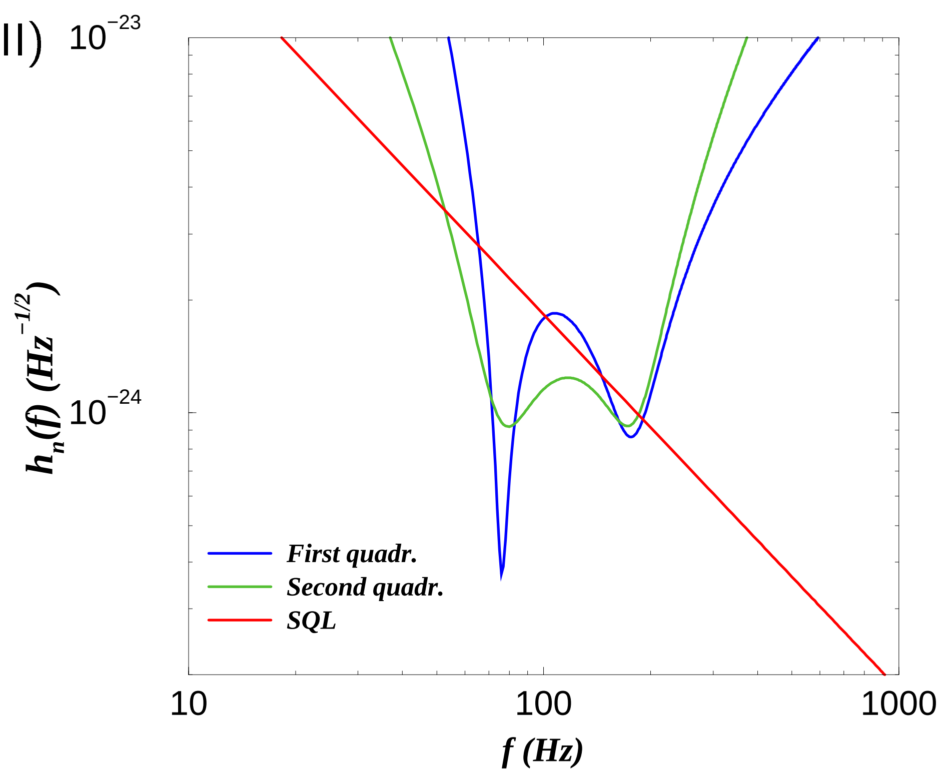
- **With high light power is crucial to take into account radiation-pressure force**

  [A.B. & Y.C. '00, '01]

  $$h_n^{\rm LIGOII}/h_{\rm SQL} \simeq 0.5 \to \mathcal{V} = 8$$

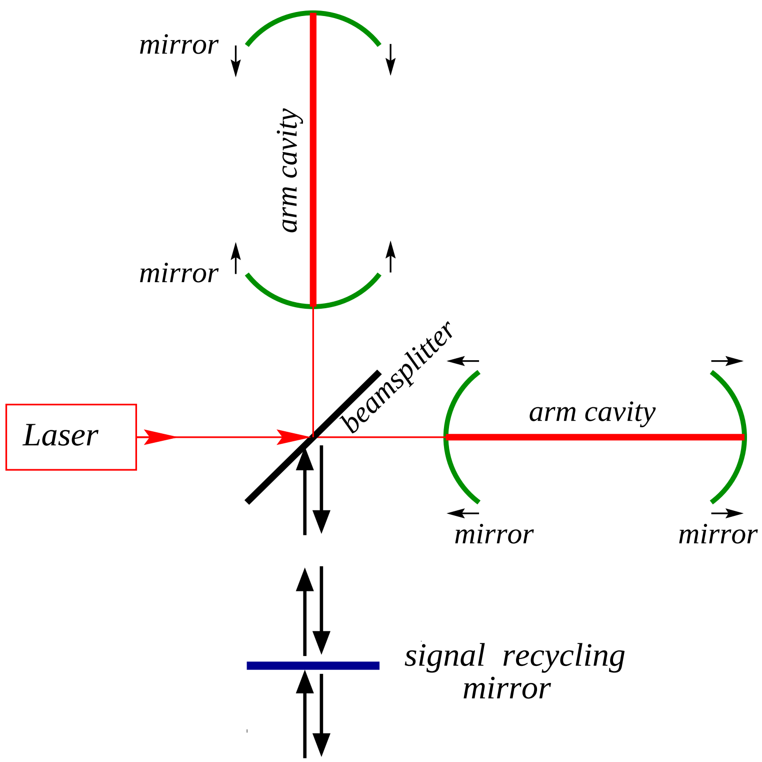
  over band of $\Delta f \sim f$

$\mathcal{V}$ $\to$ increase in the volume of the universe that can be searched for a source



Signal recycled interferometer

Initial motivation for low laser-power configuration: optical resonance due to detuning of the cavity, reshape the noise curve. [Drever '82, Meers '88]

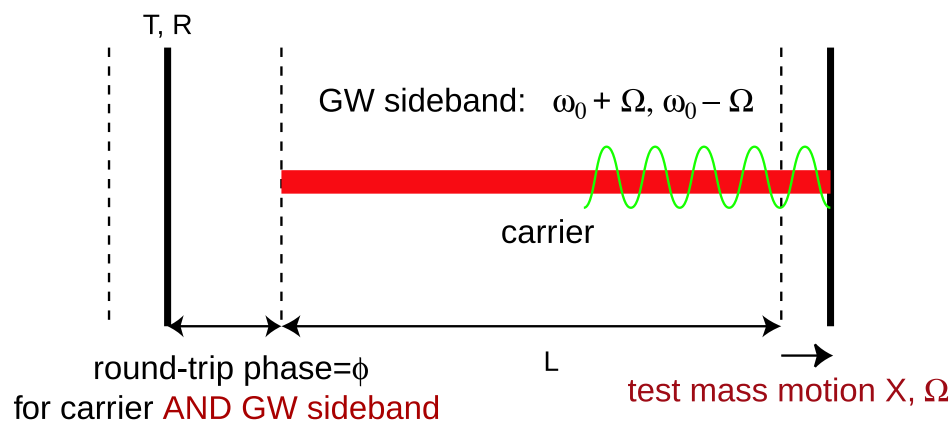


Carrier light: symmetric mode, resonate inside arm cavities, not influenced by darkport

GW sideband: antisymmetric mode, interact only with darkport. Behave differently from carrier. Resonances.

Short SR cavity: ITM and SR Cavity combined as single entity.

SR interferometer at low power: optical resonance



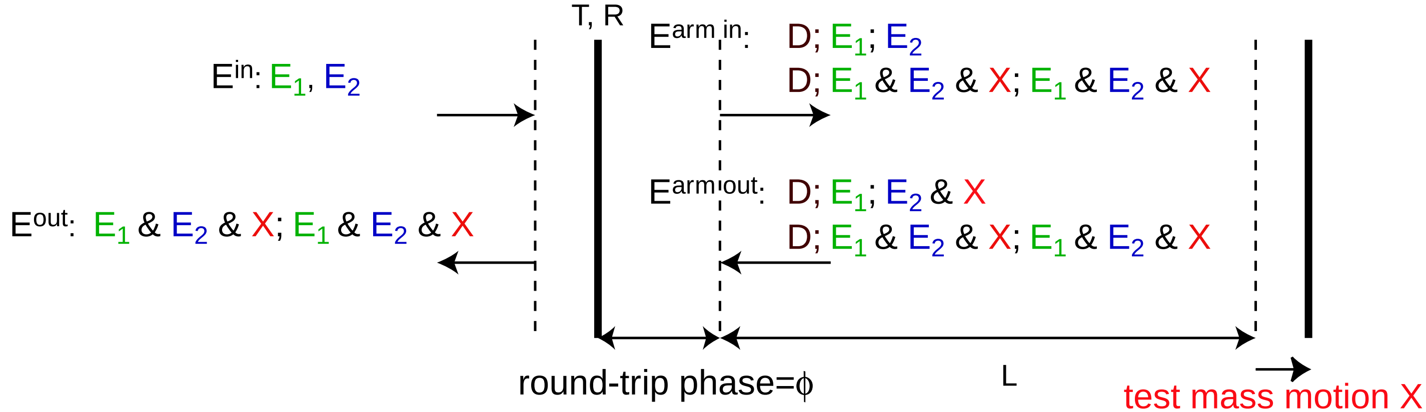
One optical resonance:

- Position: $\pm 2\Omega L/c + \phi = 2N\pi$
- Width: $Tc/(4L)$

$\Rightarrow$ Reshape noise curve:

- Tuned $(\phi = 0, \pi)$: carrier itself resonant or antiresonant . . .
- Detuned $(\phi \neq 0, \pi)$: valley in the noise curve at resonance, narrowbanding

SR interferometer with high power



Optical fields in the diagram: [Carrier; Amplitude modulation; Phase modulation]

- After a round trip inside the SR "cavity":

$$\tilde{E}_1 \to \sqrt{R}(\tilde{E}_1 \cos\phi - \tilde{E}_2 \sin\phi) + \sqrt{T}E_1$$
$$\tilde{E}_2 \to \sqrt{R}(\tilde{E}_1 \sin\phi + \tilde{E}_2 \cos\phi) + \sqrt{T}E_2$$

... ⇒ "mixing" of the amplitude and phase quadratures; shot noise and radiation-pressure noise correlated!

... ⇒ radiation pressure force F depends on X,

$$\boxed{F(\Omega) = F_0(\Omega) + R_{FF}(\Omega)X(\Omega)}$$

Dynamics of the testmass

Equation of motion for antisymmetric mode ($m\,\ddot{X} = \mathrm{Force}$):

$$-m\,\Omega^2\,X(\Omega) = \mathrm{GW\ Force} + F_0(\Omega) + R_{FF}(\Omega)\,X(\Omega)$$

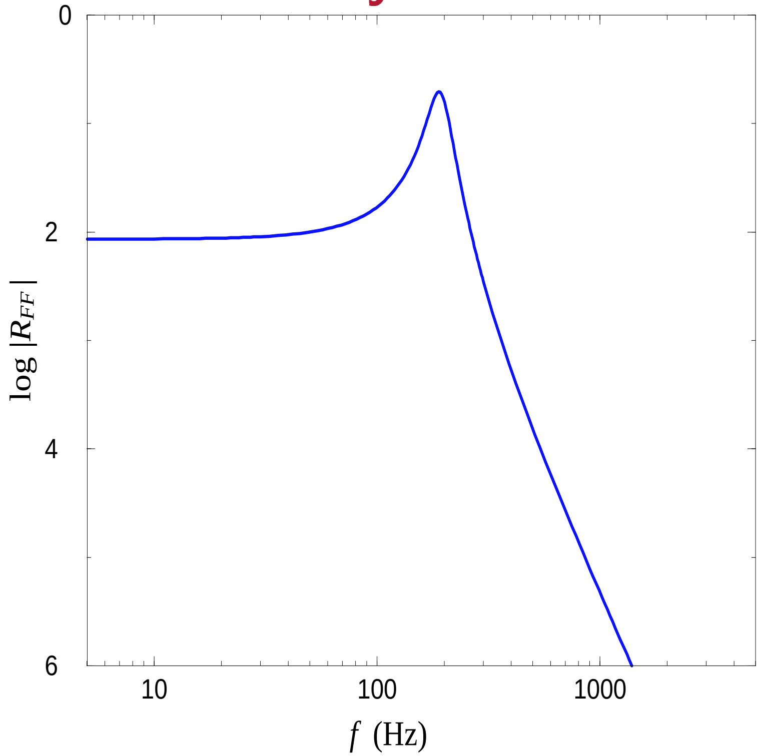
Test-mass mirrors buffeted by radiation pressure F_0, but also subject to harmonic restoring force with frequency-dependent spring constant:

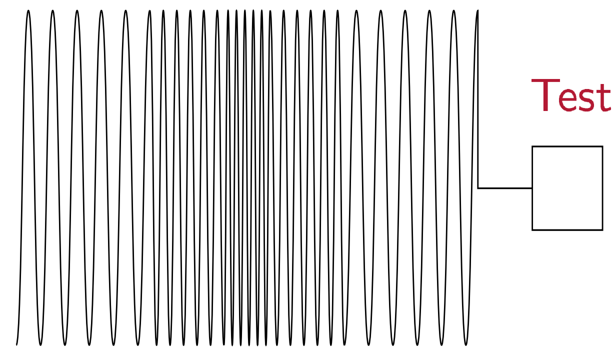
$$K_{\mathrm{spring\ constant}}(\Omega) = -R_{FF}(\Omega) \propto I_c \times (\mathrm{SR\ reflectivity}) \times (\mathrm{SR\ detuning})$$

$$\overbrace{\left[-m\,\Omega^2 - R_{FF}(\Omega)\right]}^{\text{resonances!}}\,X(\Omega) = \mathrm{GW\ Force} + F_0(\Omega)$$

Mechanical resonances get modified: no longer free test-mass!

Dynamics of the interferometer: Optical Spring

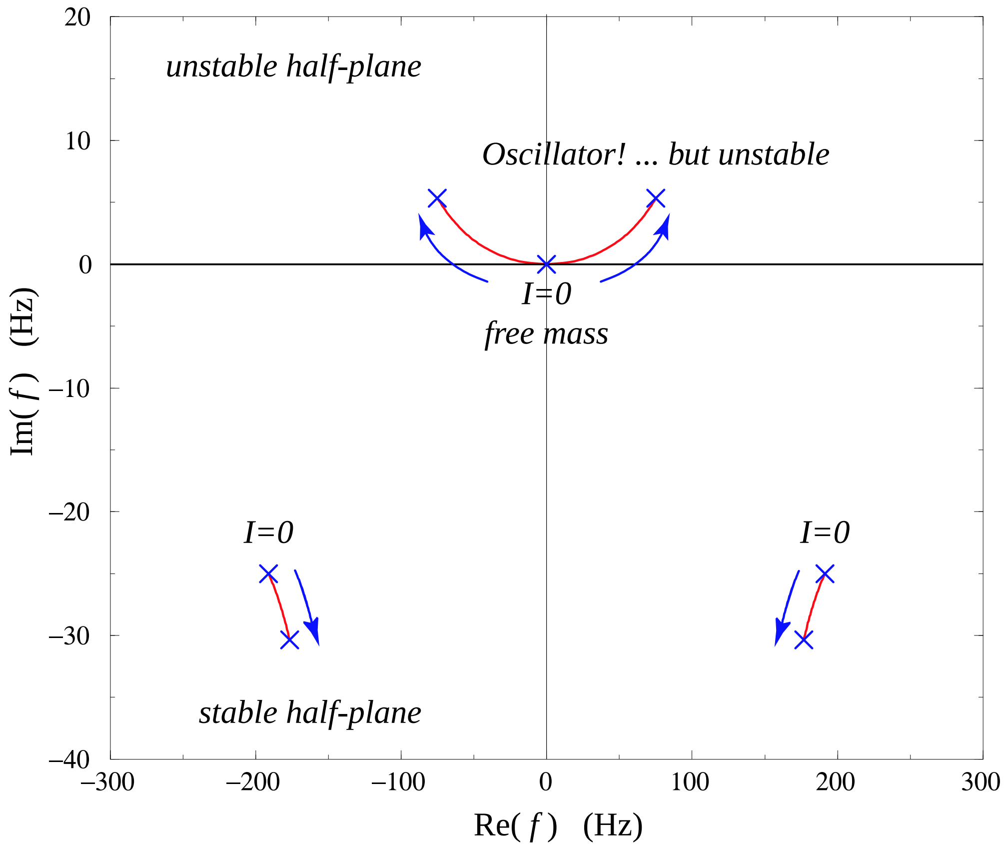




- Test mass moves at low frequency ⇒ linear restoring force
- Test mass moves at high frequency ⇒ "spring" internal mode excited

Gains sensitivity at these two resonances!

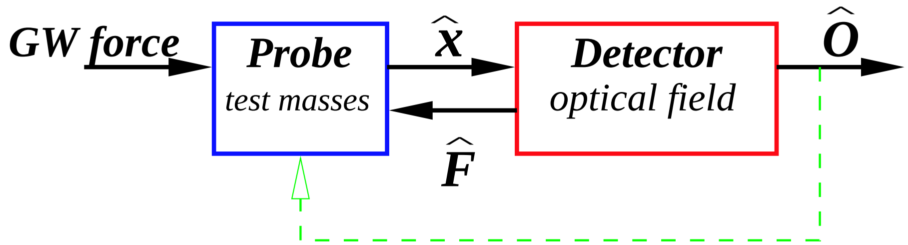
The four poles on the complex plane



I_c: 0 — 600 kW

Instability, control system and read-out schemes

One of resonant frequencies is unstable!



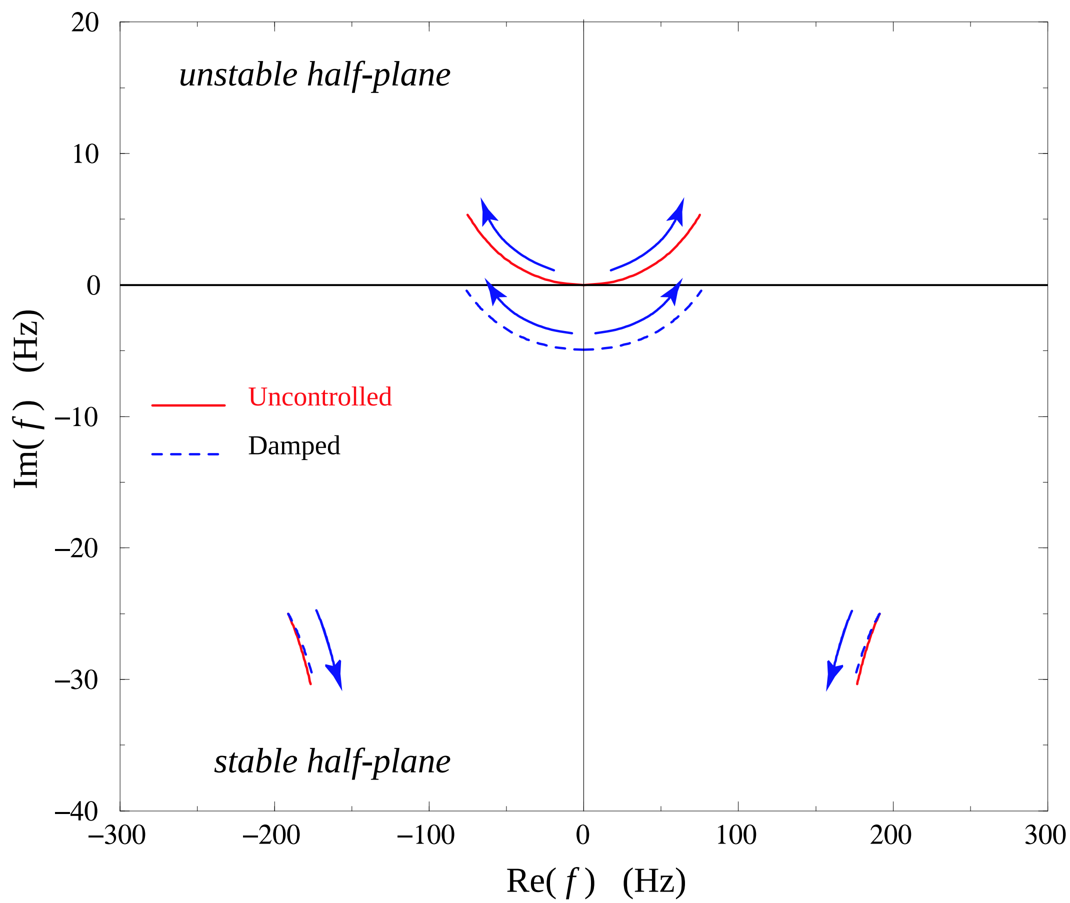
$$\hat{C} = K_C \, \hat{O}$$

Noise spectral density unchanged but our example is realistic for an-all optical control loop ...

The choice of readout scheme is crucial for optimization.

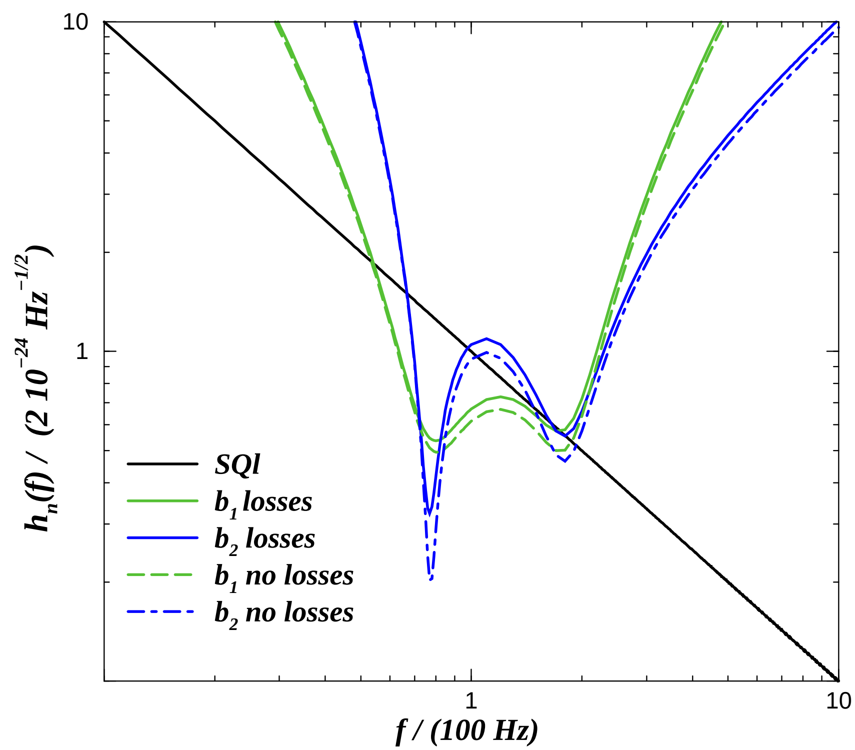
[A.B., Y.C. & Mavalvala, work in progress]

The four new poles on the complex plane



Inclusion of losses

- Loss coefficient in arm-cavity round trip $\sim 200 \times 10^{-6}$
- Fraction of photon lost at each bounce off SR mirror $\sim 2\%$
- Photodetection efficiency $\sim 90\%$



Signal-to-noise ratio (GWs from BBHs):

$$(S/N)^2_\zeta = 4 \int_0^\infty |h_{\rm GW}(f)|^2 / S_h^\zeta(f) df$$

for $\rho = 0.9$, $\phi = \frac{\pi}{2} - 0.47$, $I_o = 10^4$ W

Fractional loss in S/N for inspiraling binaries:

8% (for b_1), 21% (for b_2)

Extensions of SR interferometers

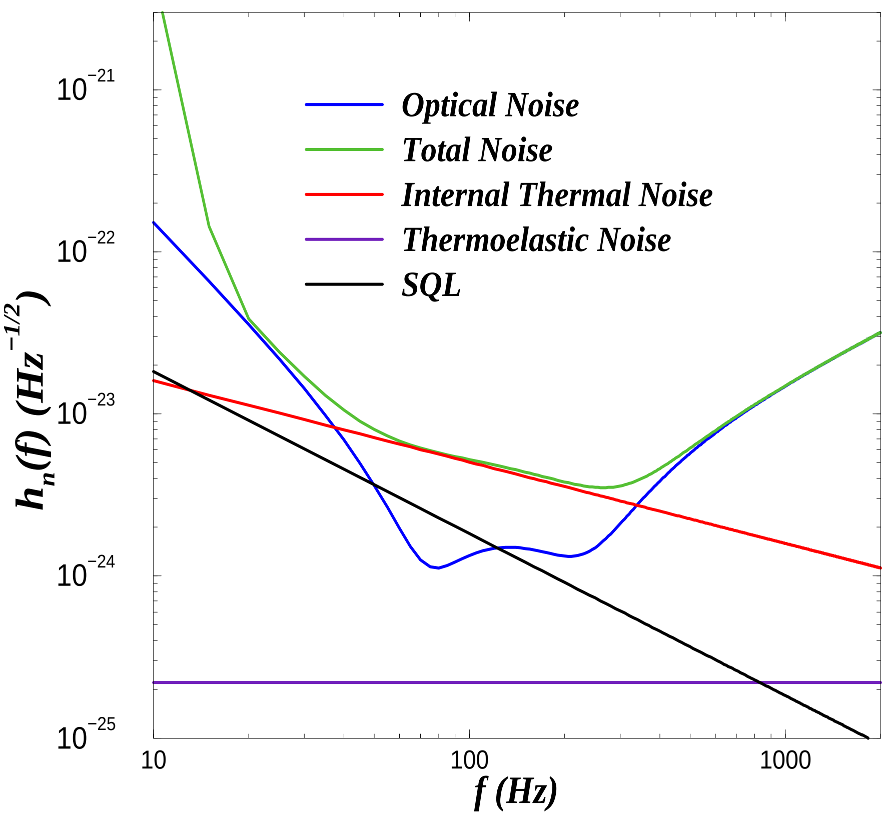
Some ideas [A.B. & Y.C., work in progress]:

- **Squeezed-input light entering the dark-port**
- **Addition of an extra mirror before SR mirror: new resonant valley in optical noise curve?**

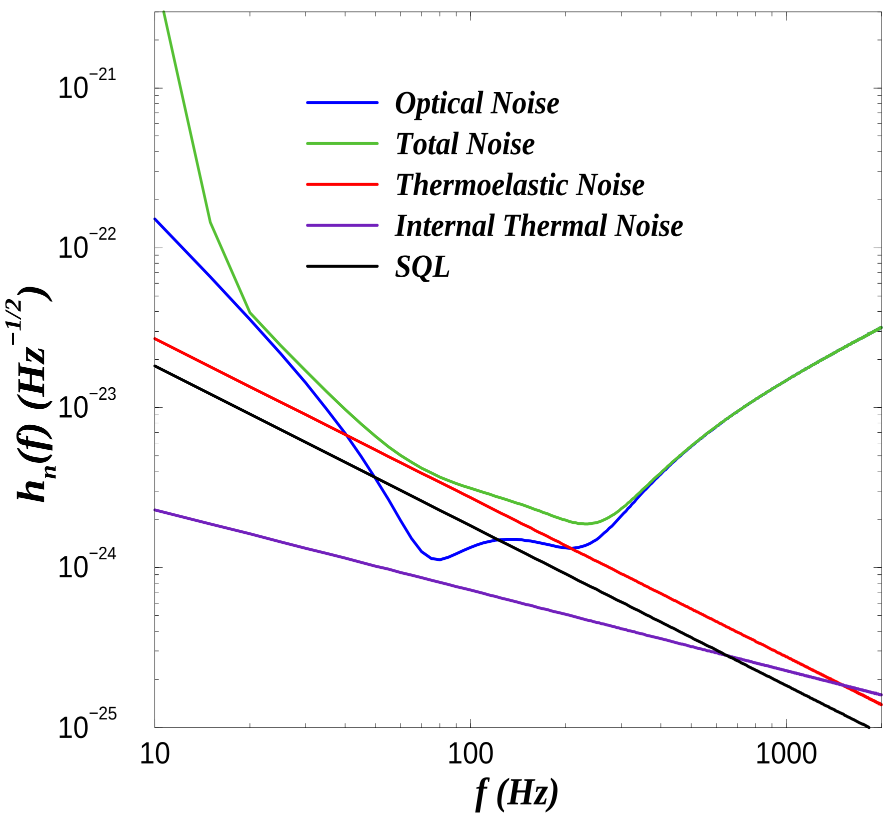
Quantum-optical noise augmented by other sources of noise

Current estimate of internal thermal, thermoelastic and seismic noises

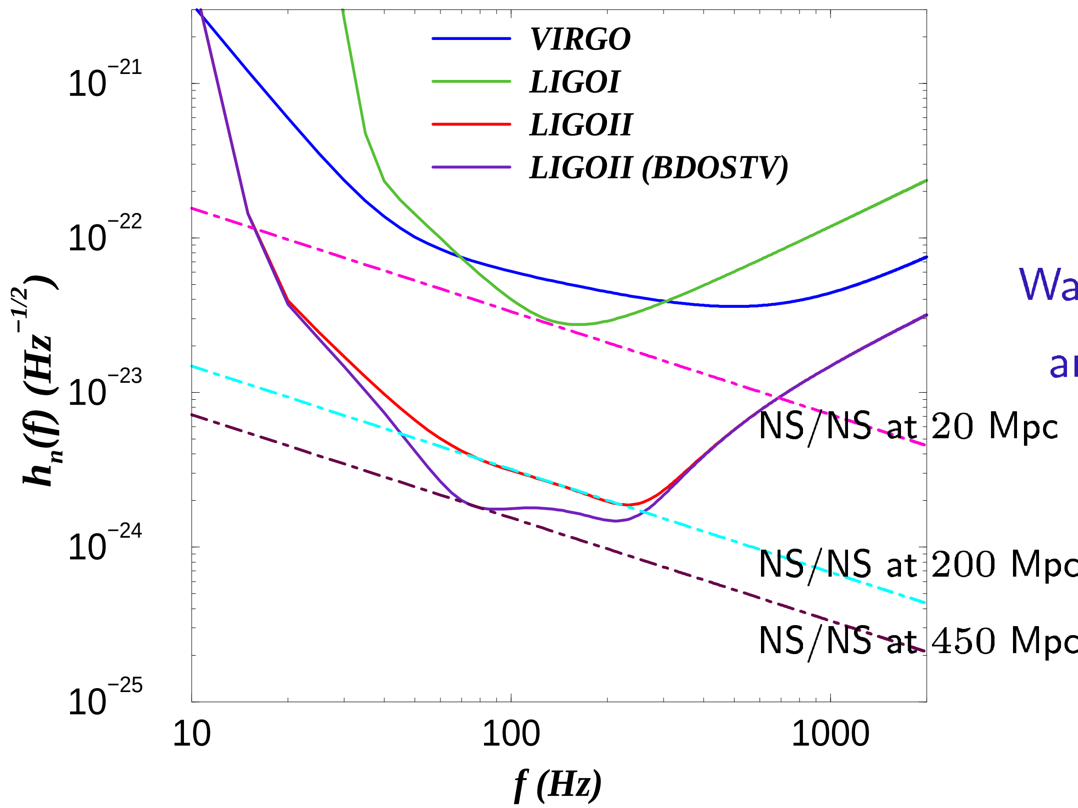
with silica mirrors



with sapphire mirrors



Lowering thermoelastic noise



Warning from mirrors' coating and internal thermal noise

By using flattened mirrors and modes

$S_h^{\rm therm} = 0.3\, S_{\rm SQL}$ NS/NS range increased from 200 Mpc to 450 Mpc

[Braginsky, D'Ambrosio, O'Shaughnessy, Strigen, Thorne & Vyatchanin, in preparation]

How to improve at low frequency (~ $10 - 10^2$ Hz)

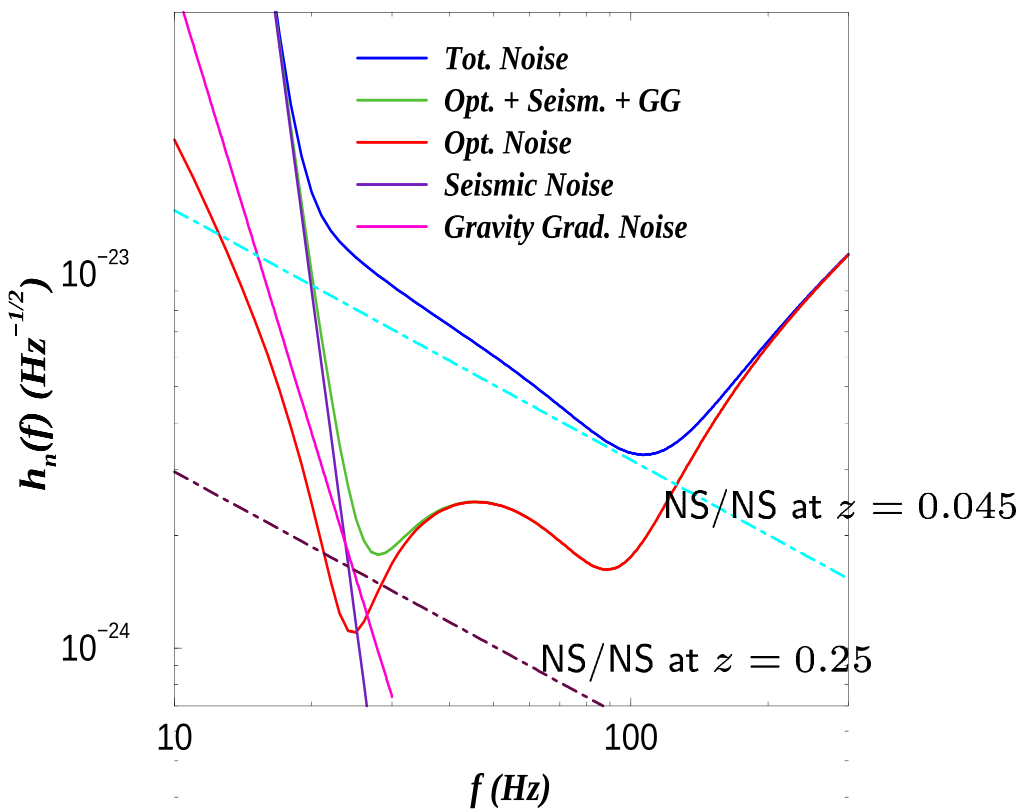
- **Thermal noise**
  - **Cryogenic techniques**

  (TAMA, Glasgow, ...)

- **Radiation-pressure noise**
  - **Larger mirror masses** $\sim 100 - 200$ **Kg**
  - **Low laser power**

- **Seismic noise**

- **Seismic gravity-gradient noise**



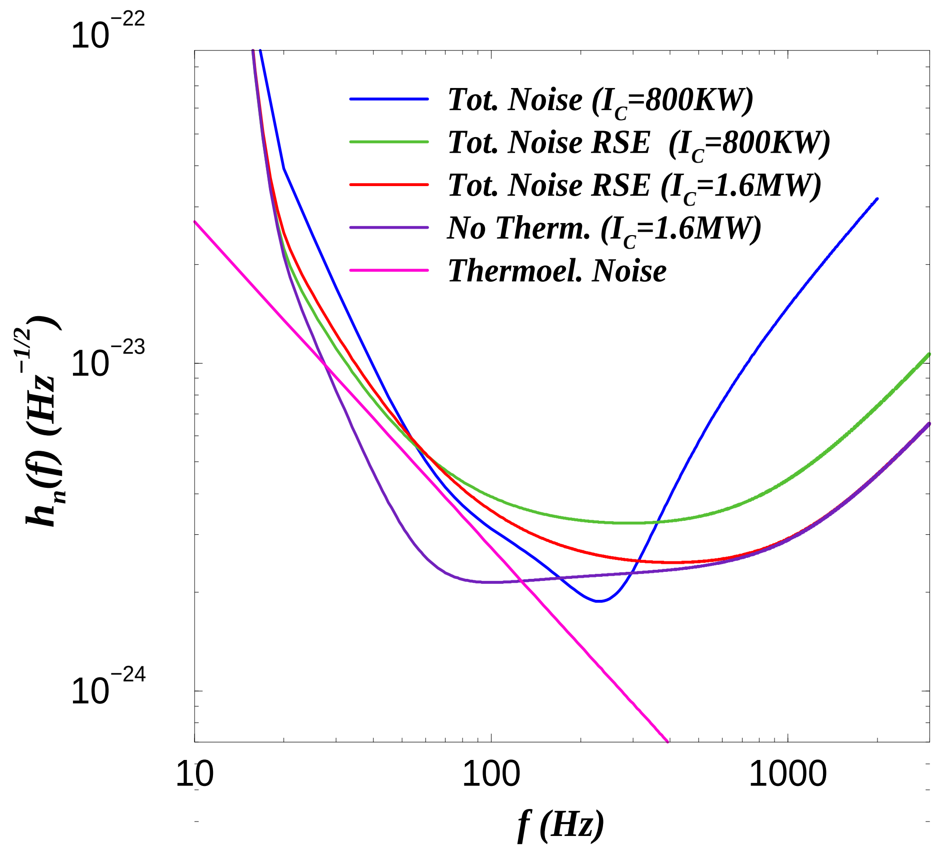
$m = 200$ Kg ! $I_c = 200$ KWatt

[Weiss '72, Saulson '84, Hughes & Thorne '98, Cella & Cuoco '98]

How to improve at high frequency ($\sim 10^2 - 3 \times 10^3$ Hz)

- **Decreasing shot noise by increasing laser light circulating in arm cavities**
  - **Low coating and substrate apsorption**

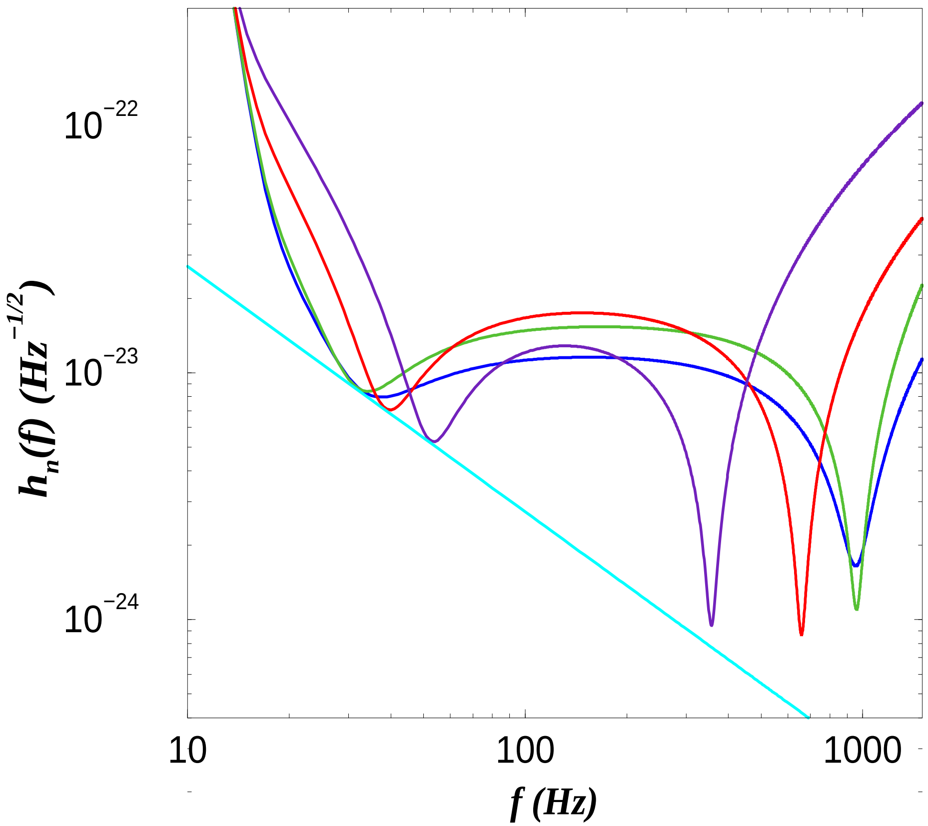
    (Glasgow, Iowa, Stanford, Syracuse, ...)
- **Narrowband configurations ...**



$h_n \sim 4.3 \times 10^{-24}/\sqrt{\mathrm{Hz}}$ at 960 Hz

$h_n \sim 2.9 \times 10^{-24}/\sqrt{\mathrm{Hz}}$ at 960 Hz

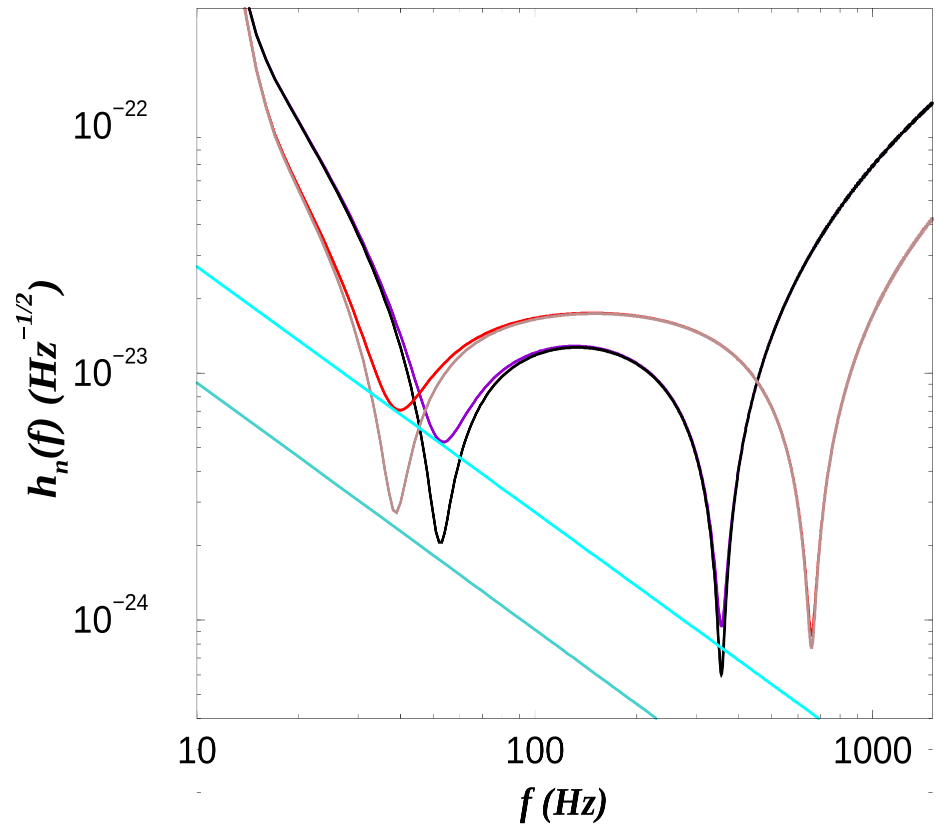
Narrowband configurations



$h_n \sim 1.4 \times 10^{-24}/\sqrt{\mathrm{Hz}}$ at 960 Hz

$h_n \sim 9 \times 10^{-25}/\sqrt{\mathrm{Hz}}$ at 660 Hz

Getting rid of thermoelastic noise ...



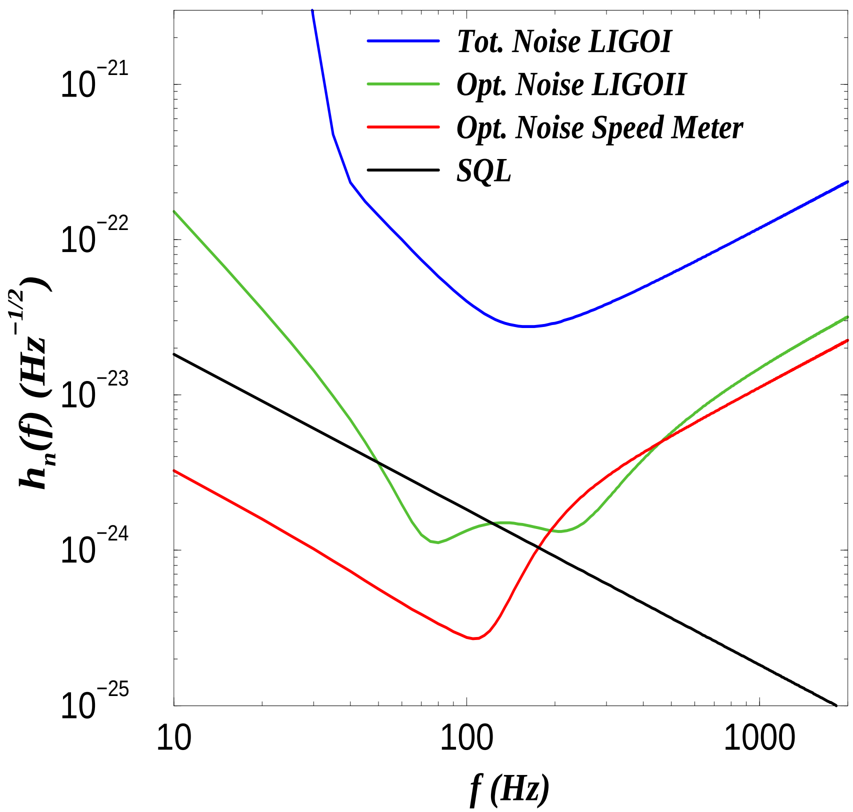
Different design: speed meter

Output signal proportional to the relative speeds of test masses rather than relative positions

[Braginsky, Gorodetsky, Khalili & Thorne '99]

New optical topologies

[Purdue 01; Purdue & Y.C., in preparation]



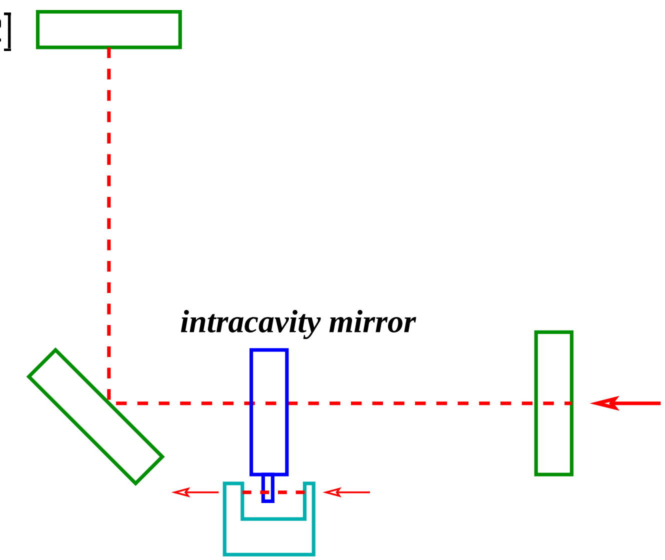
Radically different design: intracavity readout schemes

Radical redesigns of interferometers aimed at achieving perfomances below the free-mass SQL <u>but</u> with circulating power no higher than 1 MWatt.

Braginsky's group in Moscow

E.g., by using smaller test mass in final step of measurement ...

[Braginsky, Gorodetsky & Khalili '97; Braginsky & Khalili '99; Khalili '02]



Homodyne detection

$$E_{\rm output}(t) \propto \cos\omega_0 t \left[B_1 + \int_0^{+\infty} (b_1\, e^{-i\,\Omega\,t} + {\rm h.c.}) \frac{d\Omega}{2\pi}\right]$$

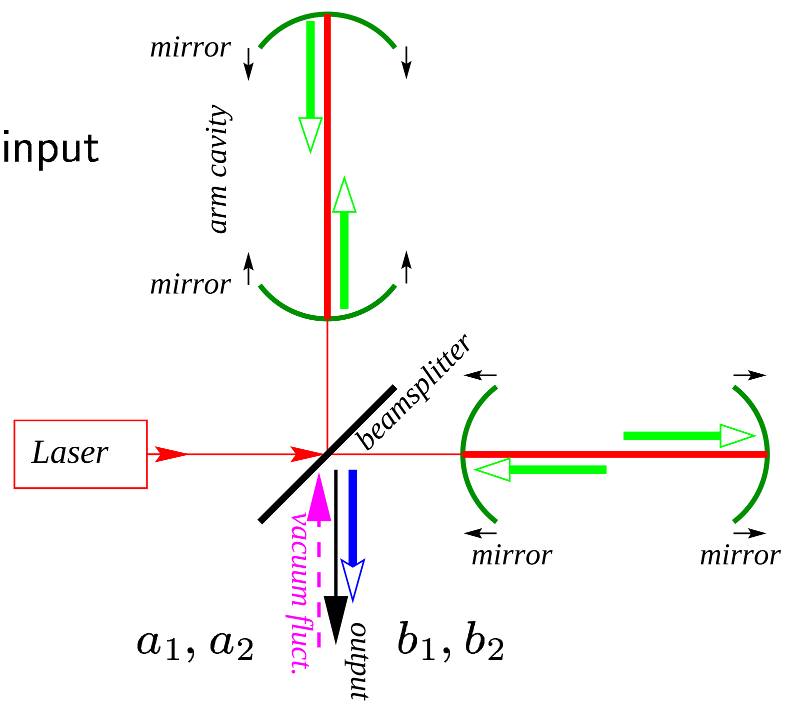
$$+ \sin\omega_0 t \left[B_2 + \int_0^{+\infty} (b_2\, e^{-i\,\Omega\,t} + {\rm h.c.}) \frac{d\Omega}{2\pi}\right]$$

The output quadratures b_1 and b_2 are functions of input quadratures a_1 and a_2 and GW signal

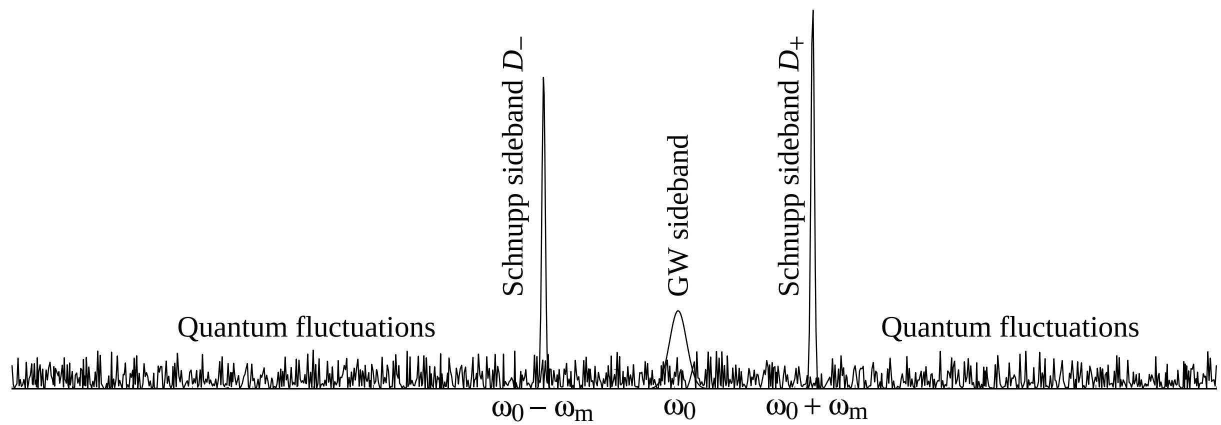
$i_{\rm PD} \propto E^2_{\rm output}(t)$ photodetector current

$$i_{\rm PD} \propto \sin\zeta\, b_1(\Omega) + \cos\zeta\, b_2(\Omega)$$

$$\sin\zeta \equiv \frac{B_1}{\sqrt{B_1^2+B_2^2}} \qquad \cos\zeta \equiv \frac{B_2}{\sqrt{B_1^2+B_2^2}}$$



RF modulation-demodulation



- The input laser at frequency ω_0 is phase modulated before entering they interferometer generating two sidebands at $\omega_0 \pm \omega_m$
- These two sidebands propagate to the dark-port output and can be used as local oscillators
- Additional noise as compared to homodyne readout scheme is introduced during photodetection process due to vacuum fluctuations at frequency bands around $\omega_0 \pm 2\omega_m$

RF modulation-demodulation

The local oscillator is needed to detect linearly the GW signal

$$O(t) = \overbrace{L(t)}^{\text{local oscillator}} + \overbrace{S(t)}^{\text{signal+vacuum fluct.}}$$

$$i_{\rm PD} \propto L(t)\, S(t)$$

The signal is no longer located around ω_0 but around $\omega_0 + \omega_m$

Demodulation: $i_{\rm PD} \times \cos(\omega_m\, t + \phi_D) \qquad \phi_D = 0, \pi/2$