

# Ph237 - Gravitational Waves

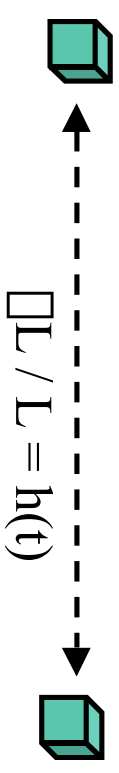
## *Week 1: Overview*

Kip S. Thorne, Caltech, 7 & 9 January 2001

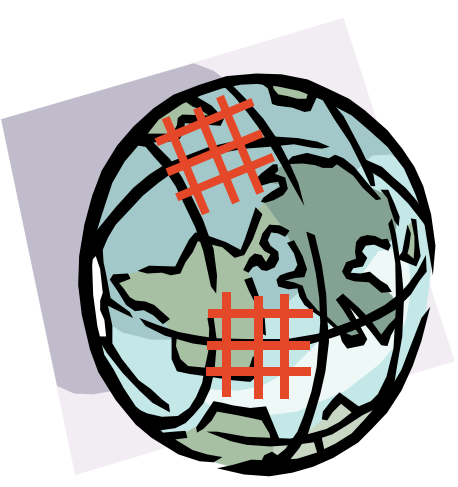
Via video feed from Cambridge England

# Physical Nature of Gravitational Waves - 1

- Waves push freely floating objects apart and together
  - Local inertial frames do not mesh



- Like non-meshing of Cartesian coordinates on Earth's surface
  - Earth's curvature causes non-meshing
- Spacetime curvature causes inertial-frame non-meshing



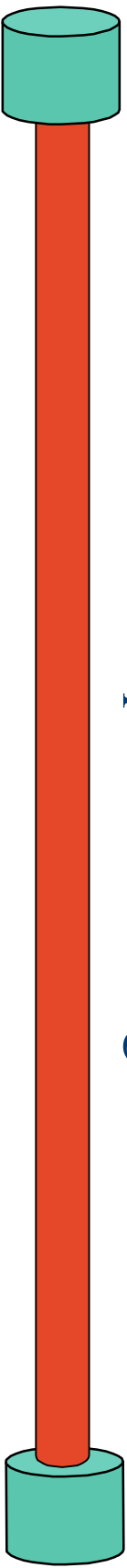
- *Gravitational waves are ripples of spacetime curvature*

# Physical Nature of Gravitational Waves - 2

- Great richness to a wave's spacetime curvature:
  - Heuristically:
    - Stretch and squeeze of space
    - Slowing and speeding of rate of flow of time
    - ...



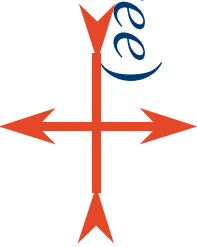

– Measure stretch and squeeze with light beams



- Does light wavelength get stretched and squeezed the same as mirror separation, so no effect is seen?
  - NO! Spacetime curvature influences light differently from mirror separations.
- Mathematically:
- Curvature described by rank-4 Riemann tensor,  $R_{\alpha\beta\gamma\delta}$

# Physical Nature of Gravitational Waves - 3

- Stretch and squeeze are:   $\square_L / L = h$

- *transverse* to direction of propagation
- Equal and opposite along orthogonal axes (*trace-free*) 
- Force pattern invariant under  $180^\circ$  rotation
- Contrast with EM waves: invariant under  $360^\circ$  rot'n 

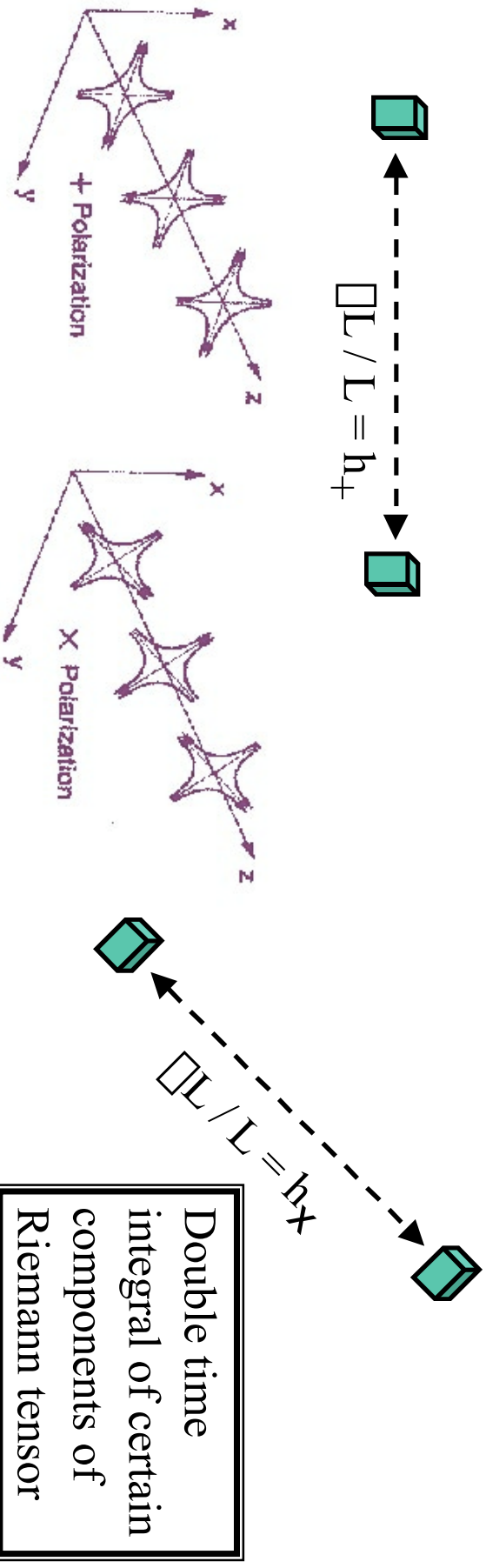
- (Spin of quantum) =  $(360 \text{ degrees}) / (\text{invariance angle})$   
 $= 1$  for photon,  $2$  for graviton
  - Irreducible representation of Little Subgroup of Lorentz grp

- Two polarizations: axes rotated  $90^\circ$  EM 

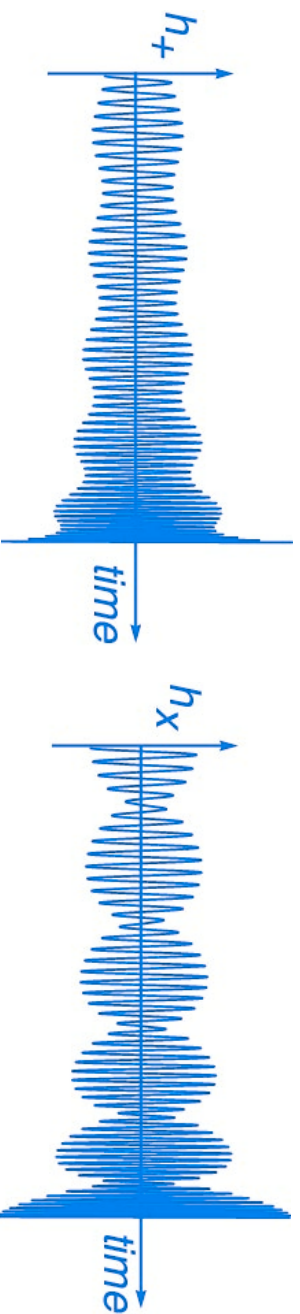
- $45^\circ$  GW  plus  cross

# Physical Nature of Gravitational Waves - 4

- Each polarization has its own gravitational-wave field



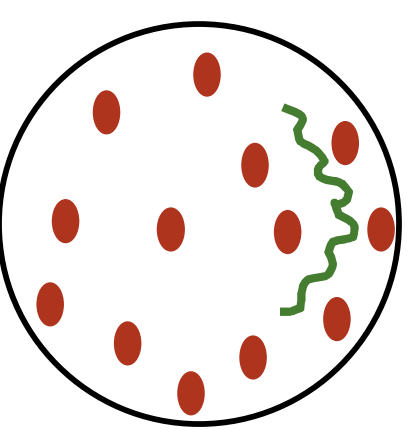
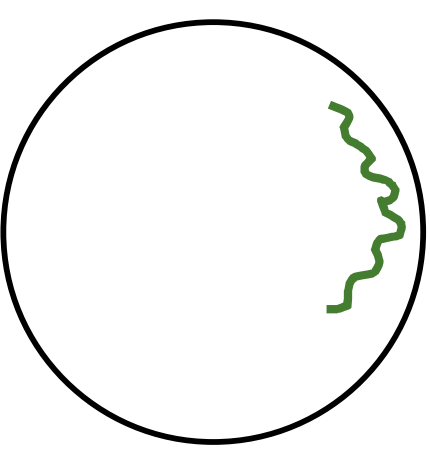
- These fields' evolutions  $h_{+}(t)$  &  $h_{\times}(t)$  are the *waveforms*



*Waveforms*  
carry detailed  
Information  
about source

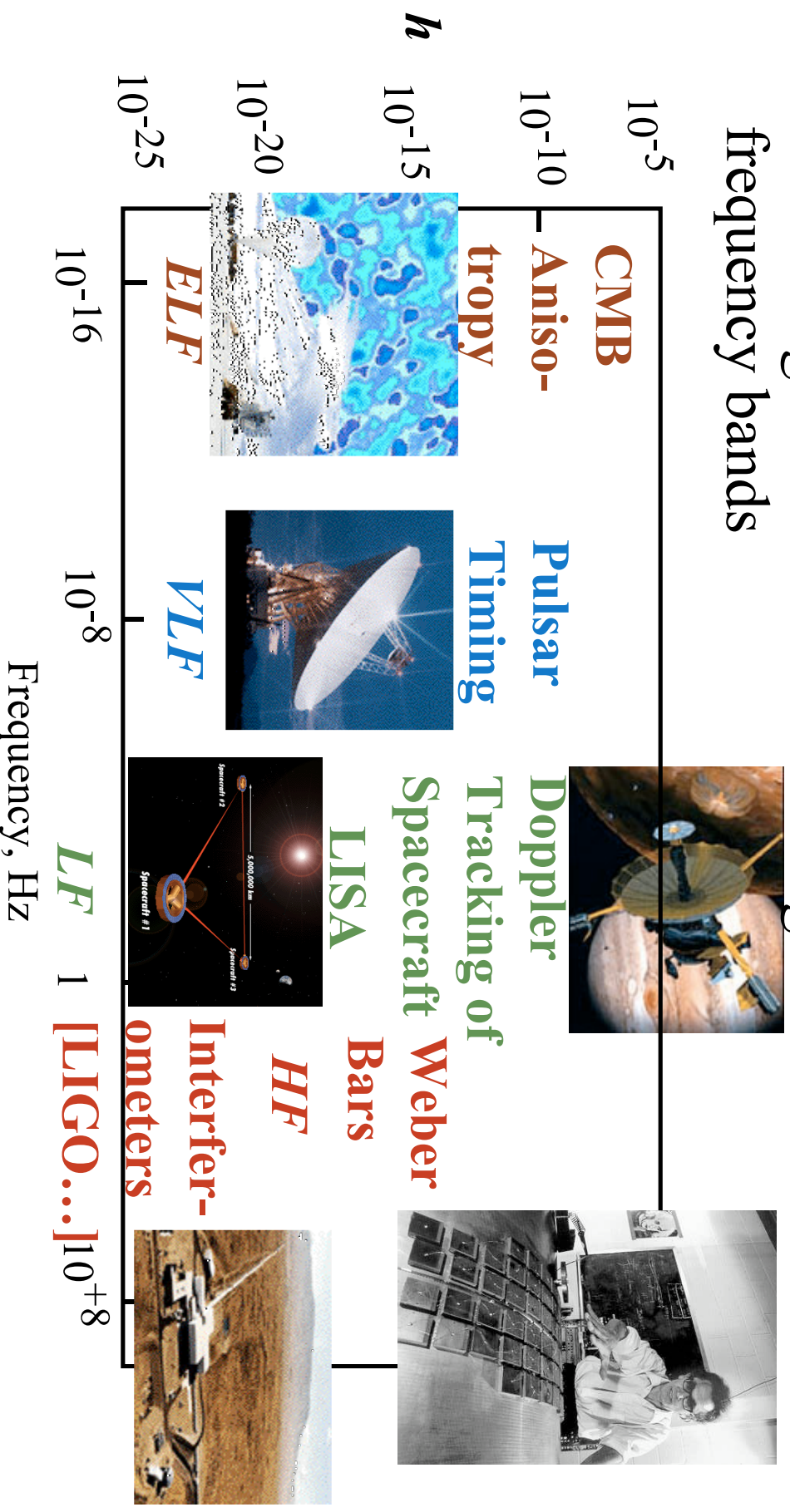
# Propagation of Gravitational Waves

- High-frequency waves (wavelength  $\lambda \ll$  radius of curvature  $R$  of background spacetime; geometric optics): propagate at light speed
  - $\Rightarrow$  graviton has rest mass zero (like photon)
  - Redshifted and grav'ly lensed, like light
- If  $\lambda \sim R$ , scattered by spacetime curvature
- Absorption by matter in our universe:
  - Negligible ... even back to big bang
- Dispersion due to interaction with matter:
  - Negligible
  - Example: Universe filled with neutron stars or black holes:
    - In propagating around the universe once:
    - Dispersion delays the GW by about one wavelength  $\lambda$



# The Gravitational Wave Spectrum

- Spectrum of known and expected sources extends over 22 decades of frequency
- Promising sensitivities are being achieved in four frequency bands



# Some Sources in our Four Bands:

<b>ELF</b>	<b>VLF</b>	<b>LF</b>	<b>HF</b>
<b>CMB</b>	<b>Pulsar</b>	<b>Doppler</b>	<b>LIGO</b>
<b>Anisotropy</b>	<b>Timing</b>	<b>LISA</b>	

---

The Big Bang Singularity in which the Universe was born, Inflation of Universe

Exotic Physics in Very Early Universe: Phase transitions, cosmic strings, domain walls, mesoscopic excitations, ... ?

Massive BH's  
(300 to 30 million  
suns),  
Binary stars  
Soliton stars?  
Naked  
singularities?

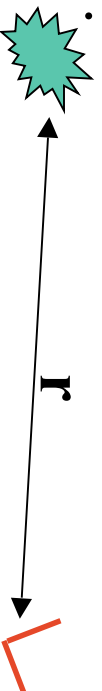
Small BH's (2 to  
1000 suns),  
Neutron stars  
Supernovae  
Boson stars?  
Naked  
singularities?



# Caltech Faculty Involved in GW Research

- LIGO (high frequencies,  $\sim 10$  Hz to  $\sim 1000$  Hz):
  - Barish, Drever, Libbrecht, Weinstein, Kip
- LISA (low frequencies,  $\sim 10^{-4}$  Hz to  $\sim 0.1$  Hz):
  - Prince, Phinney, Kip. + heavy JPL involvement
- Doppler tracking (very low frequencies)
  - Kulkarni
- Cosmic microwave polarization anisotropy
  - Kamionkowski, Lange, Readhead
- **CaJAGWR**: Caltech/JPL Association for Gravitational Wave Research
  - Seminars  $\sim$  every other Friday [alternate with LIGO seminars]
    - <http://www.cco.caltech.edu/~cajagwr/>
      - Links to LIGO, LISA, and other GW sites

# Multipolar Decomposition of Waves

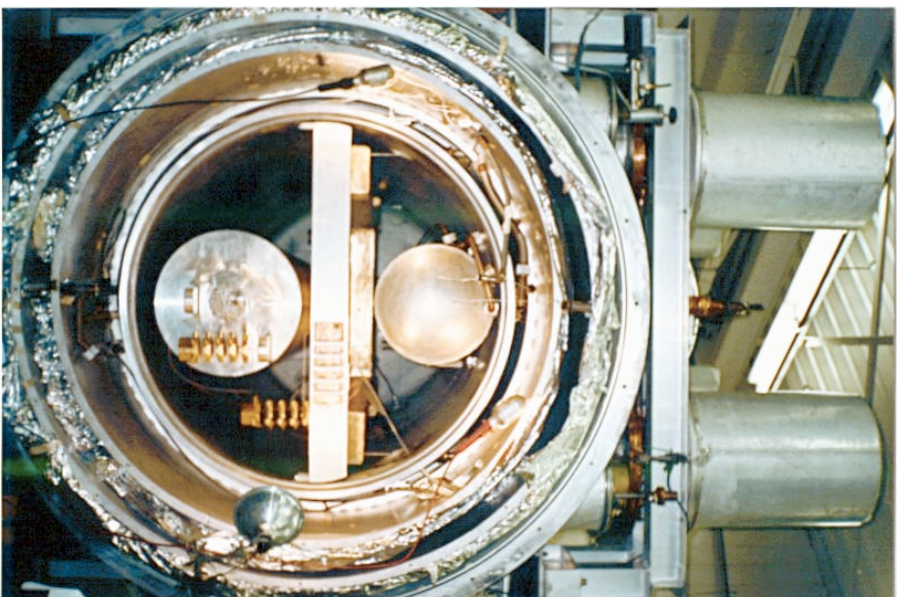
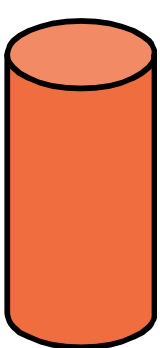


- Expand  $h$  in multipole moments of source's mass and mass-current (momentum) distributions:  $M_0, M_1, M_2, \dots; S_1, S_2, \dots$
- $h$  is dimensionless; must fall off as  $1/r \Rightarrow$ 
  - $h \sim (G/c^2)(M_0/r) \ \& \ (G/c^3)(\dot{M}_1/r) \ \& \ (G/c^4)(\ddot{M}_2/r) \ \& \ \dots$ 
    - Mass can't oscillate
    - Momentum can't oscillate
    - Mass quadrupole Moment dominates
  - $\& \ (G/c^4)(\dot{S}_1/r) \ \& \ (G/c^5)(\ddot{S}_2/r) \ \& \ \dots$ 
    - Angular Momentum can't oscillate
    - Current quadrupole
- Theorem in canonical field theory:
  - (Waves' multipole order)  $\geq$  (spin of quantum) = 2 for graviton

# Strengths of Waves

- Source: mass  $M$ , size  $L$ , oscillatory period  $P$ ,
  - quadrupole moment  $M_2 \sim M L^2$
- Quadrupole moment approximation:
  - $h \sim (G/c^4)(\ddot{M}_2/r) \sim (G/c^4)(M L^2/P^2) / r$ 
    - $\sim (G/c^4)(\text{internal kinetic energy}) / r$
    - $\sim (1/c^2)$  (Newton potential of [mass-equivalent] kinetic energy)
    - $\sim (1/c^2)$  (Newton potential of [mass-equivalent] potential energy)
- Higher multipoles: down by  $(v/c)$  to some power
- Magnitude:
  - Colliding BH's or NS's @  $r \sim 100 \text{ Mpc} \sim 3 \times 10^8 \text{ ltyr} \sim 3 \times 10^{27} \text{ cm}$
  - [Mass-equivalent] Kinetic energy  $\sim M_{\text{sun}}$
  - $h \sim \text{few} \times 10^{-22}$

# International Network of Bar Detectors Now in Operation [~1000 Hz]



Louisiana State U. - Allegro



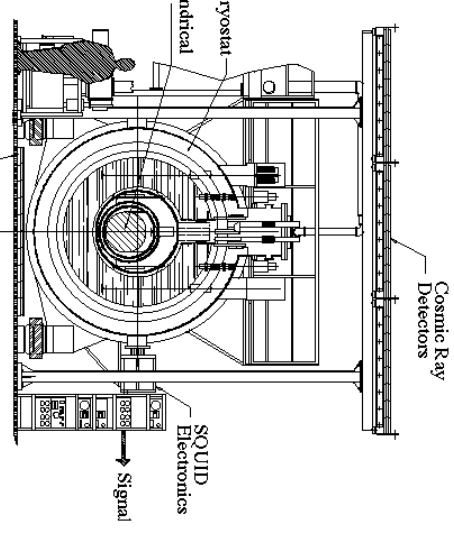
U. West Australia - Niobe



U. Padova - Auriga



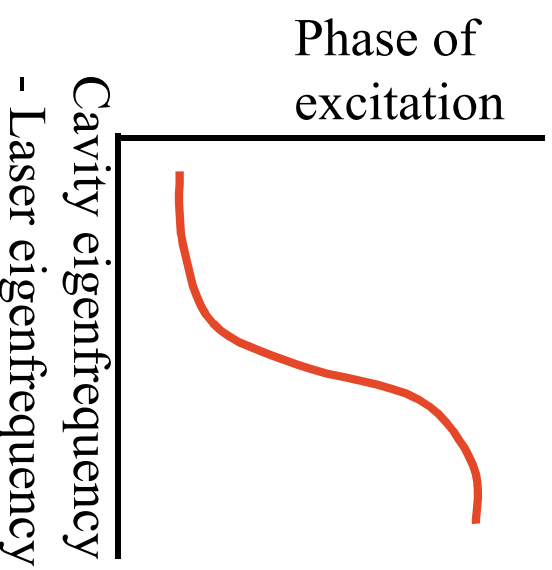
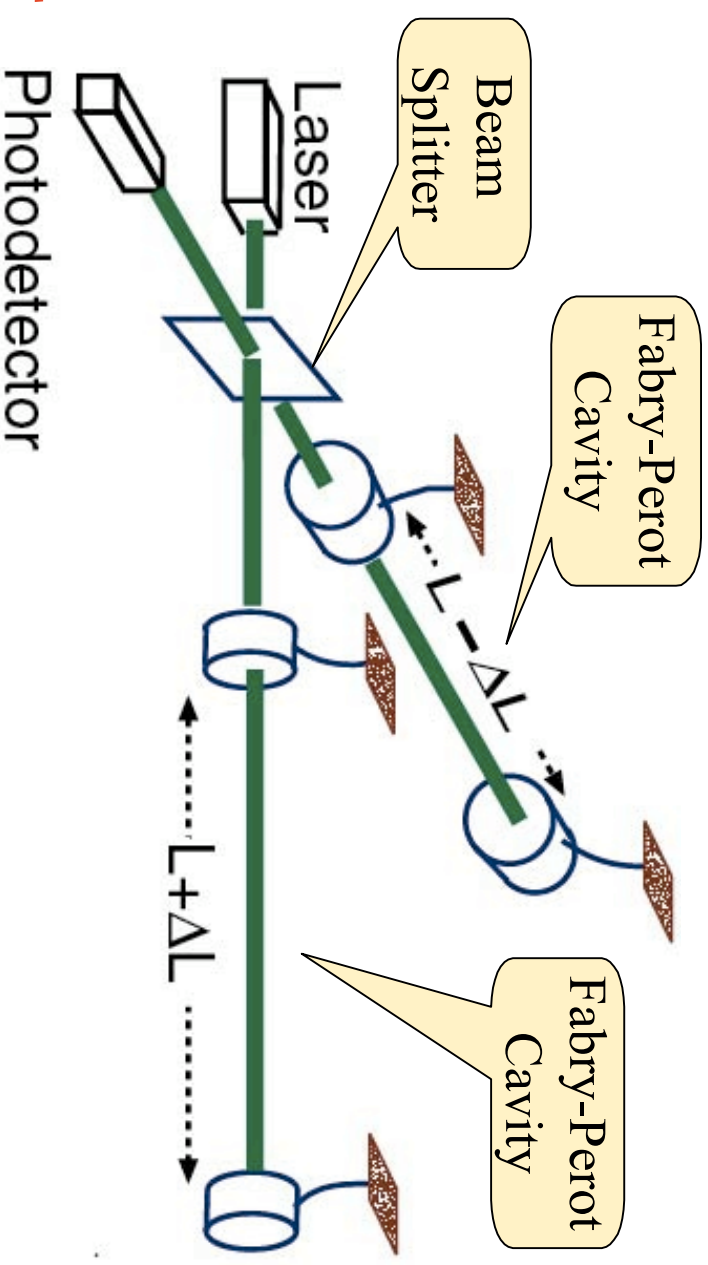
CERN - Explorer



U. Rome - Nautilus  
Cosmic Ray Detectors

# How a LIGO Interferometer Works

- **Schematic description of detector:**



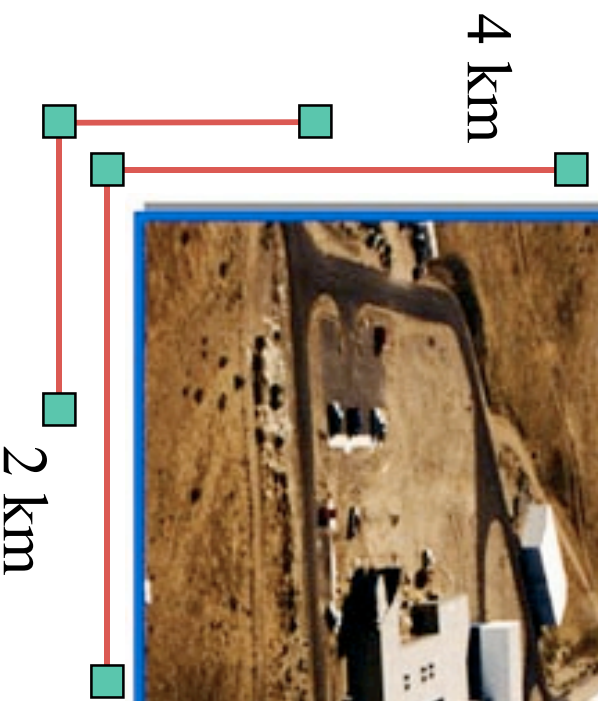
$$\Delta L = h L \approx 4 \times 10^{-16} \text{ cm}$$

$\approx 10^{-21}$       **4 km**

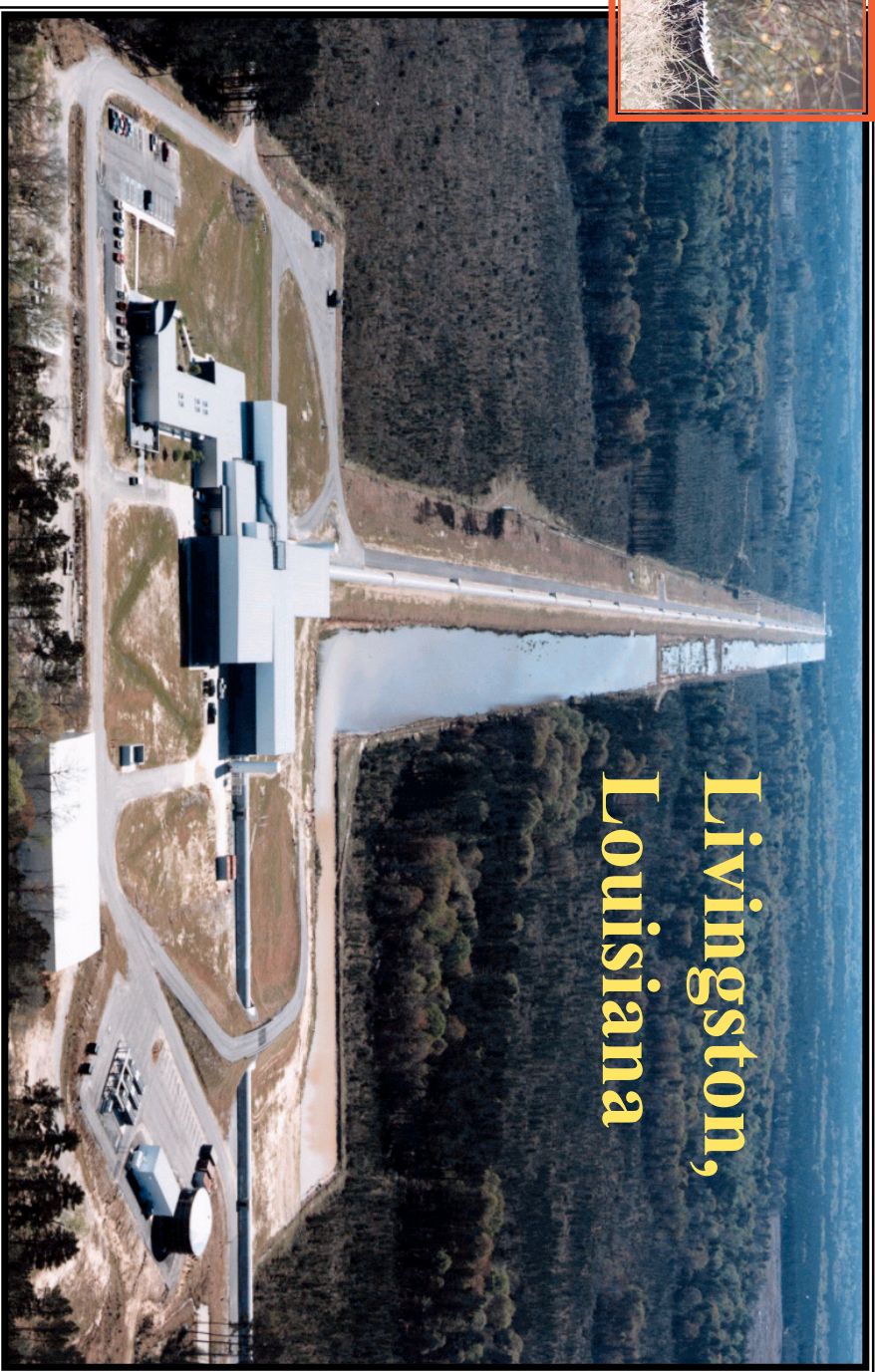
# LIGO

*Collaboration of ~350 scientists at ~30 institutions*

**Hanford Washington**



# LIGO



4 km

- *First searches for GW's: 2002 to 2006 -- sensitivity where plausible to see waves*
- *Upgrade to advanced interferometers: ~2007; 3000 higher event rate*
  - *new search: 2008 ... -- sensitivity where should see rich waves from wide variety of sources*

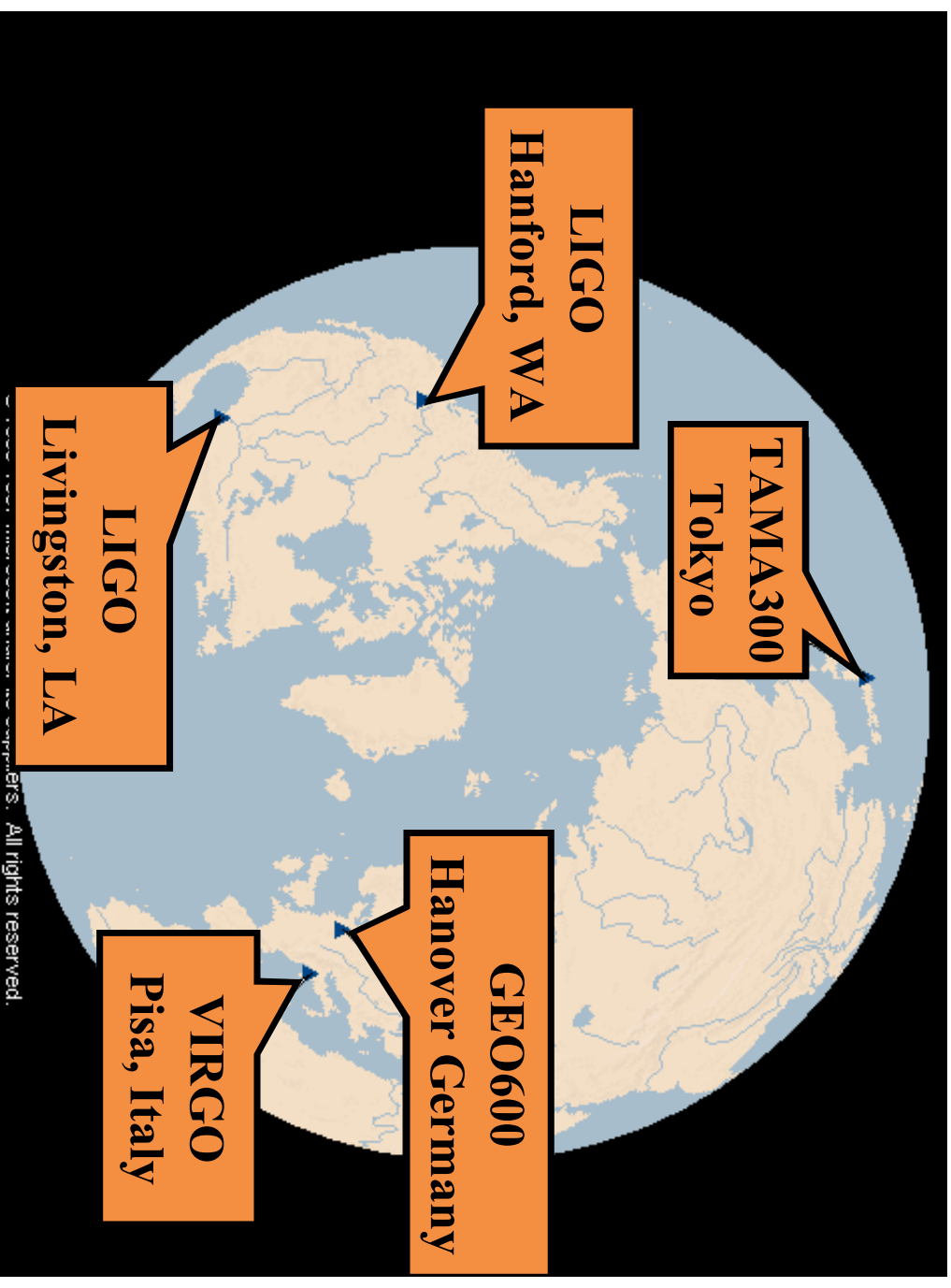
# LIGO Organization

- **LIGO Laboratory**
  - Responsible for Facilities; and for Design, Construction, & Operation of Interferometers
  - Caltech & MIT; Director: Barry Barish [Caltech]
- **LIGO Scientific Community (LSC)**
  - Formulates science goals
  - Carries out Interferometer R&D
  - ~350 scientists and engineers in ~25 institutions
    - Caltech, California State University, Carleton, Cornell, FermiLab, U. Florida, Harvard, Iowa State, JILA (U. Colorado), LSU, Louisiana Tech, MIT, U. Michigan, U. Oregon, Penn State, Southern U., Stanford, Syracuse, U. Texas-Brownsville, U. Wisconsin-Milwaukee, ACIGA (Australia), GEO600 (Britain & France), IUCAA (India), NAOJ-TAMA (Japan), Moscow State U. & IAP-Nizhny Novgorod (Russia)
  - Spokesman: Rai Weiss [MIT]



# International Network of Interferometric Detectors

- **Network Required for:**
  - **Detection Confidence**
  - **Waveform Extraction**
  - **Direction by Triangulation**



# LIGO's International Partners



VIRGO: Pisa, Italy [Italy/France]



GEO600, Hanover Germany [UK, Germany]



TAMA300, Tokyo [Japan]



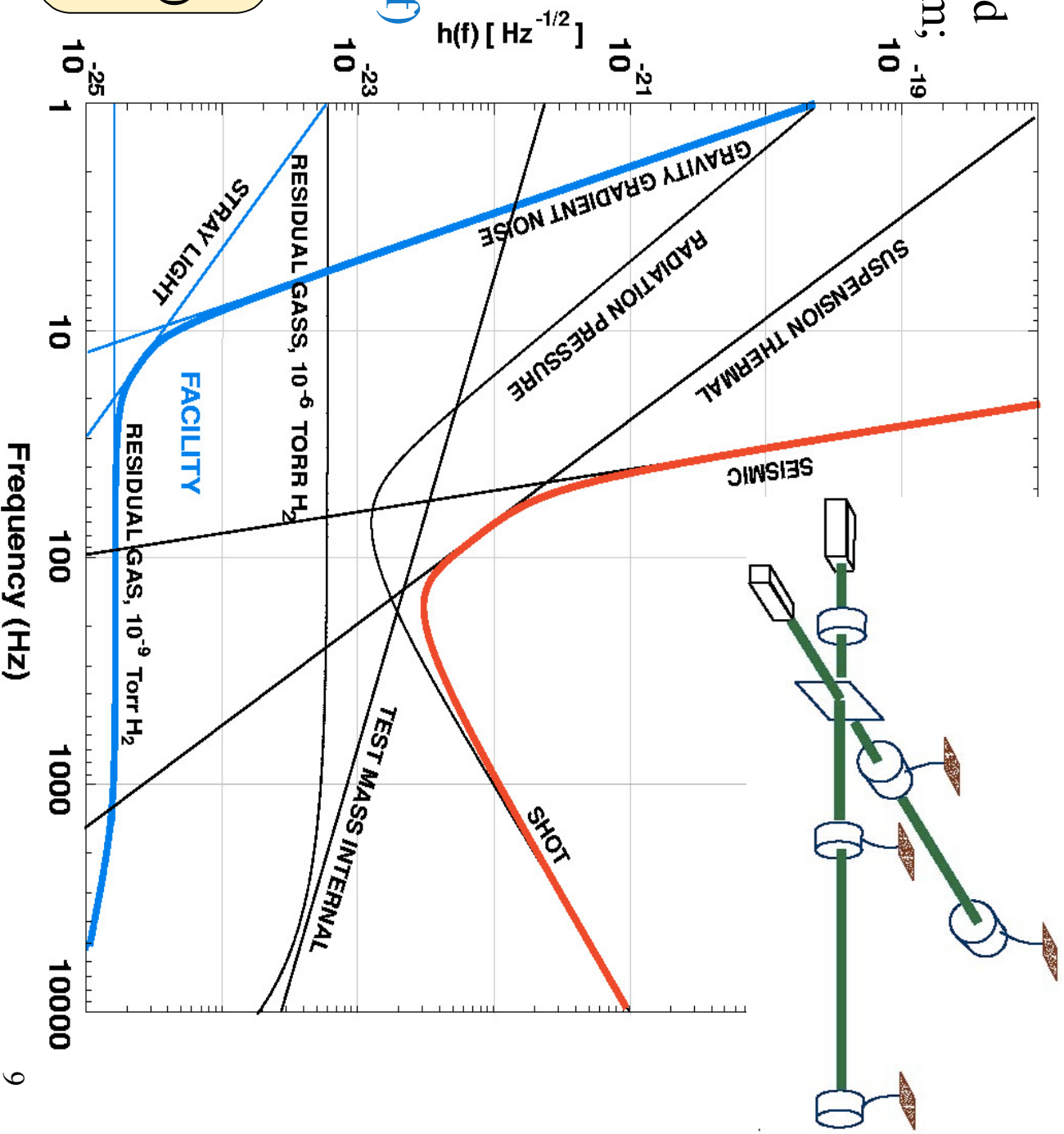
AIGO, Jin-Jin West Australia

# LIGO: Initial Interferometers

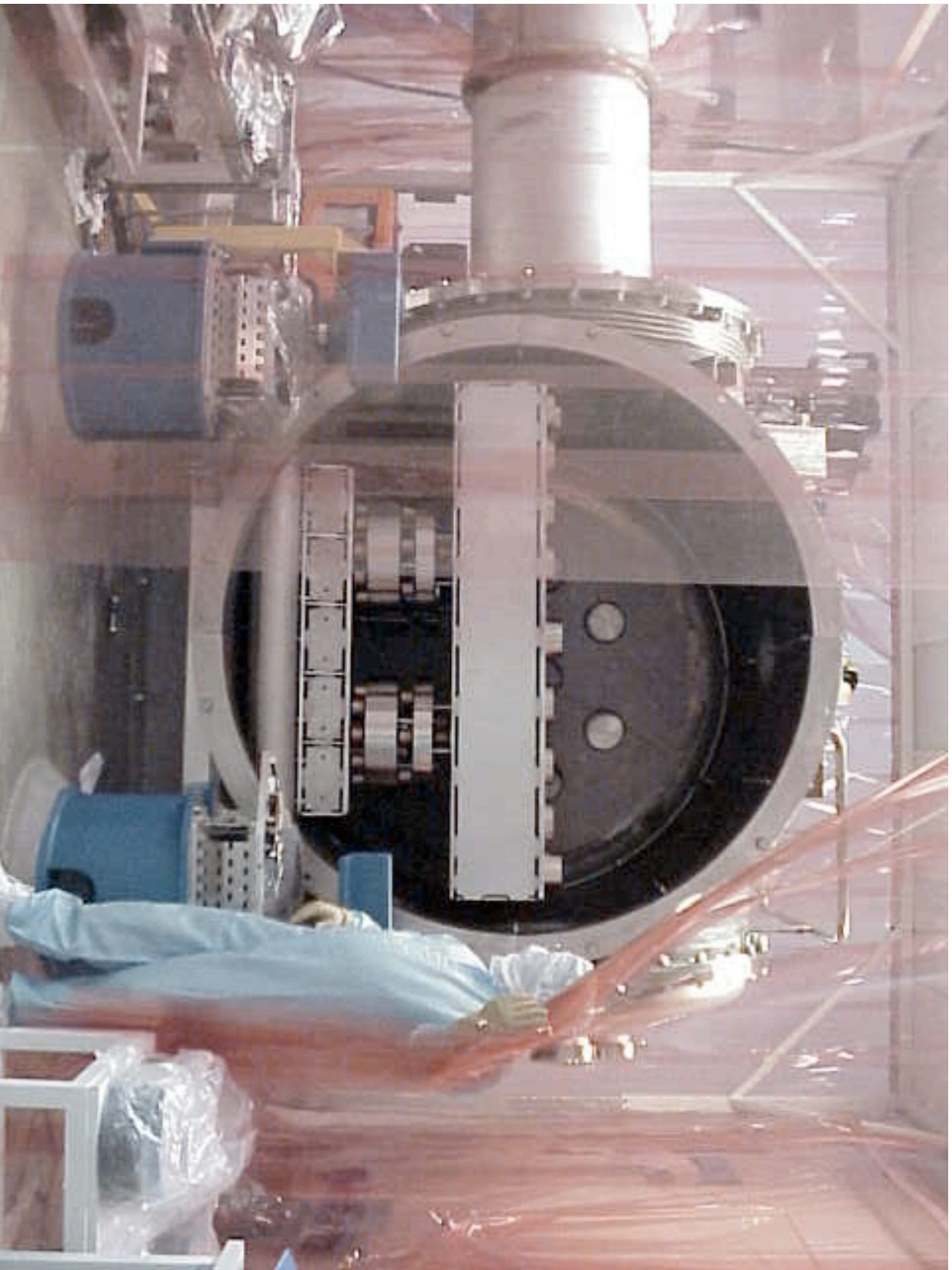
- Have been installed (Hanford 4km, 2km; Livingston 4 km)
- Are being debugged; first search underway (at poor sensitivity)

$$h_{\text{rms}} = h(f) \quad f \approx 10 h(f)$$

Square root of Spectral density of  $h(t)$  [“theory of random processes”]



# Seismic Isolation



# Test-Mass Mirror and its Suspension



# Mirror Installation and Alignment

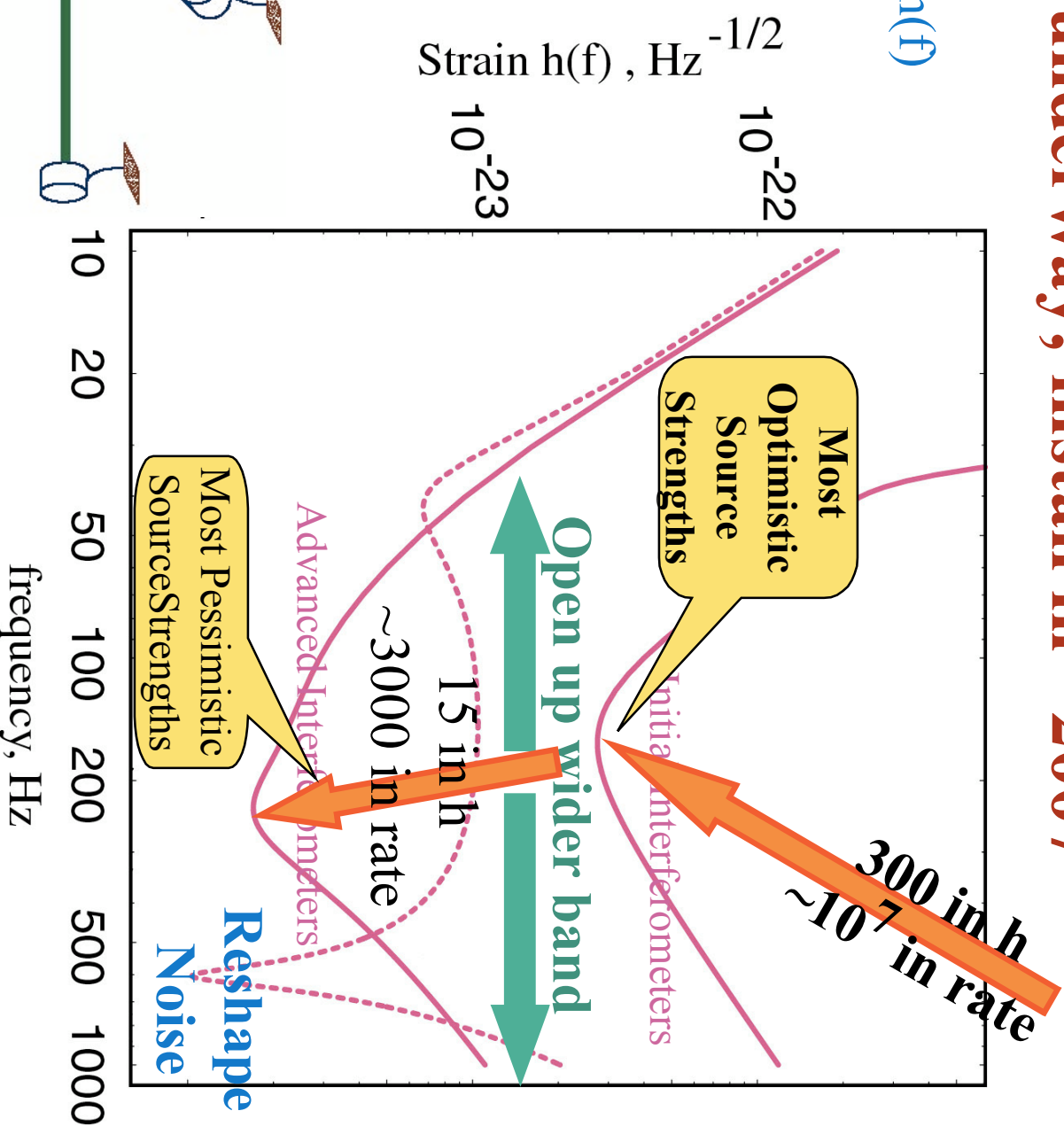
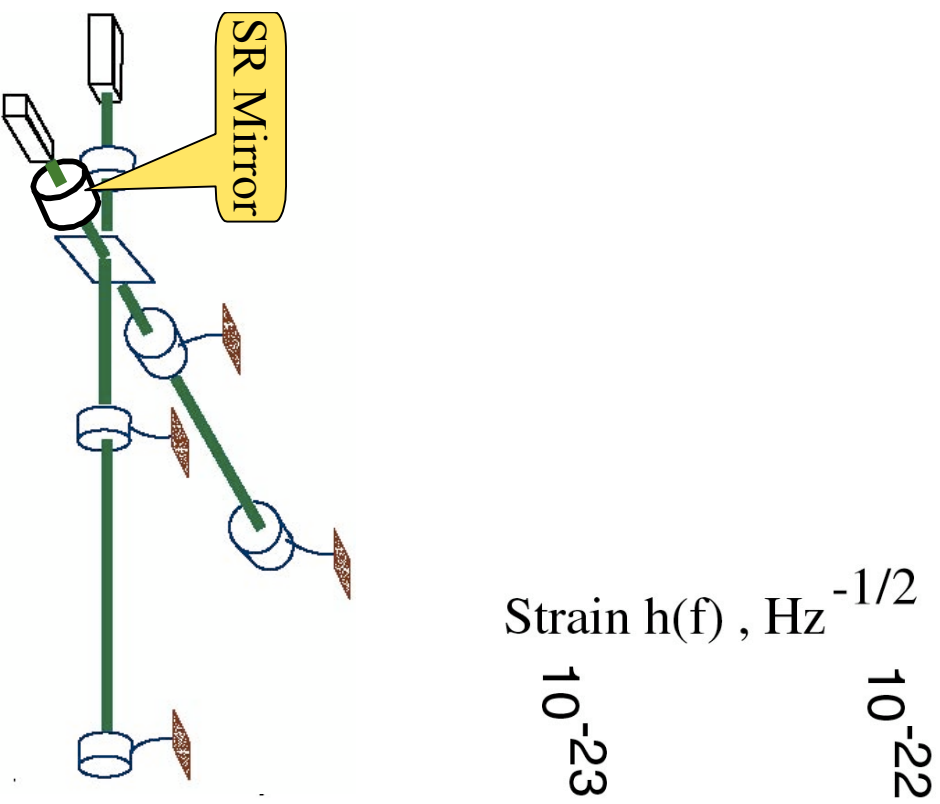


# Protection from Elements



# LIGO: From Initial Interferometers to Advanced R&D underway; install in ~2007

$$h_{\text{rms}} = h(f) \quad f \approx 10 h(f)$$



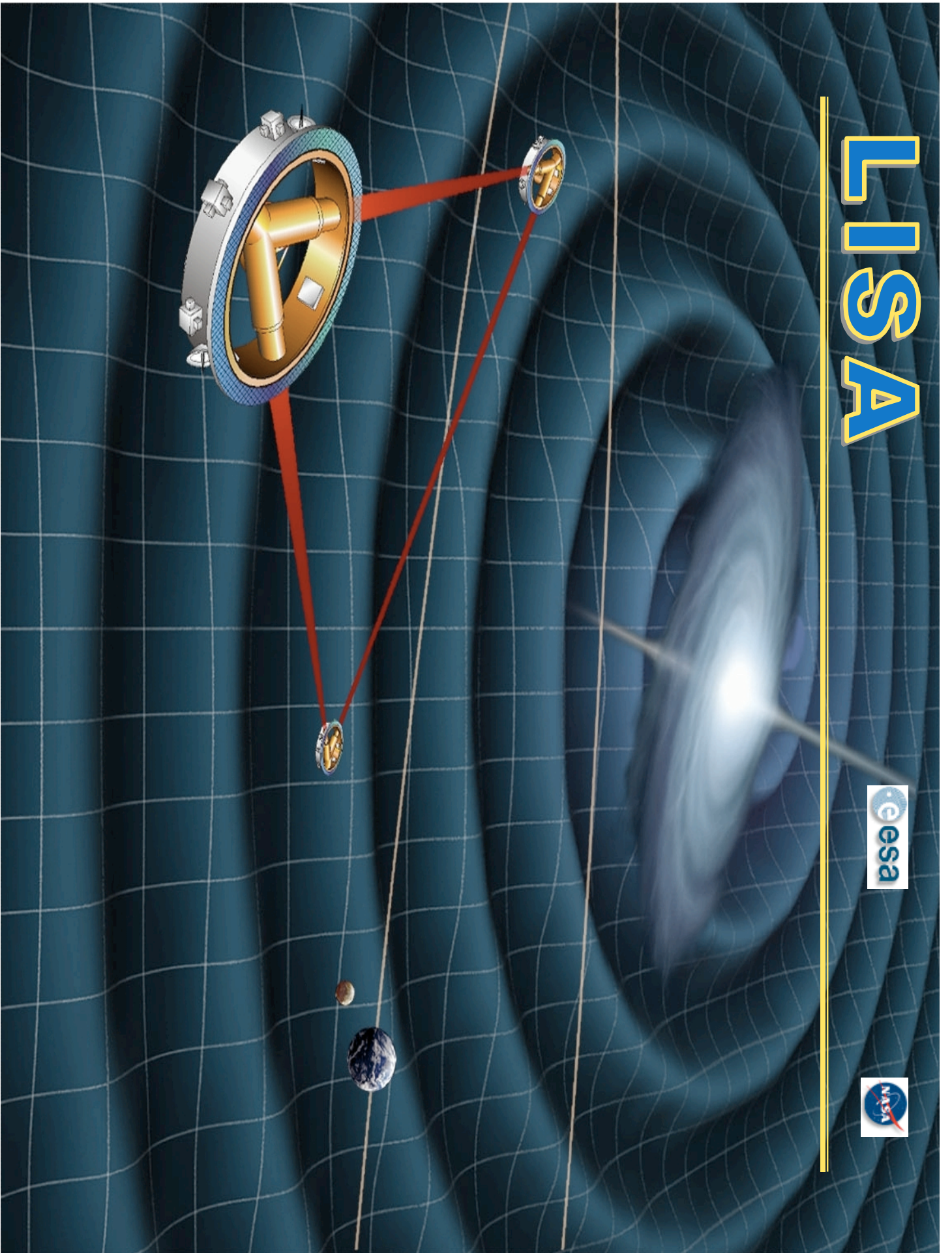


# Advanced IFOs: The Technical Challenge

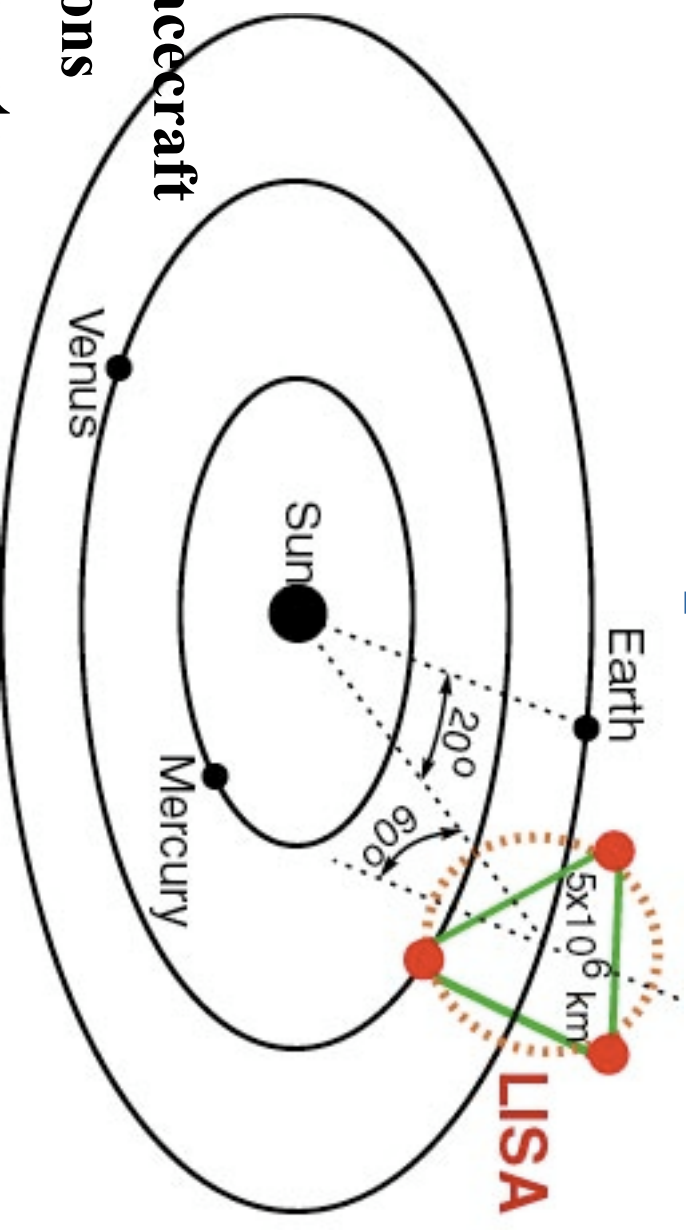
- **In advanced interferometers:**
  - Monitor motions of 40 kg sapphire mirrors to:
    - $\sim 10^{-17}$  cm  $\sim 1/10,000$  diameter of atomic nucleus
    - $\sim 10^{-13}$  of the wavelength of light
    - $\sim$  the half width of the mirror's quantum wave function
- **Quantum Nondemolition (QND) Technology**
  - Branch of quantum information science



# LISA

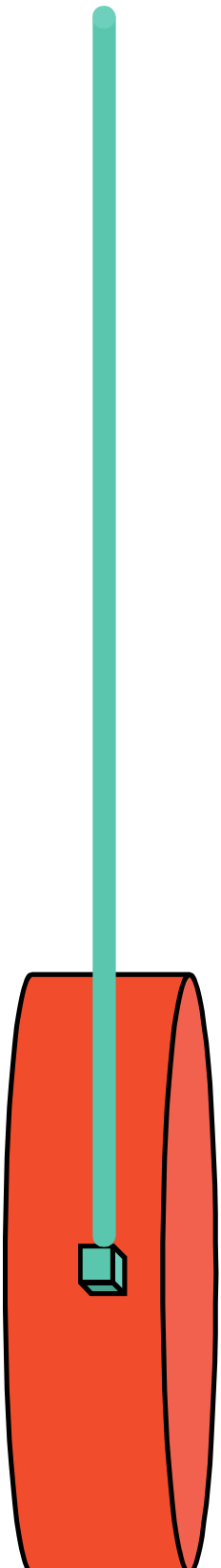


# LISA: Laser Interferometer Space Antenna



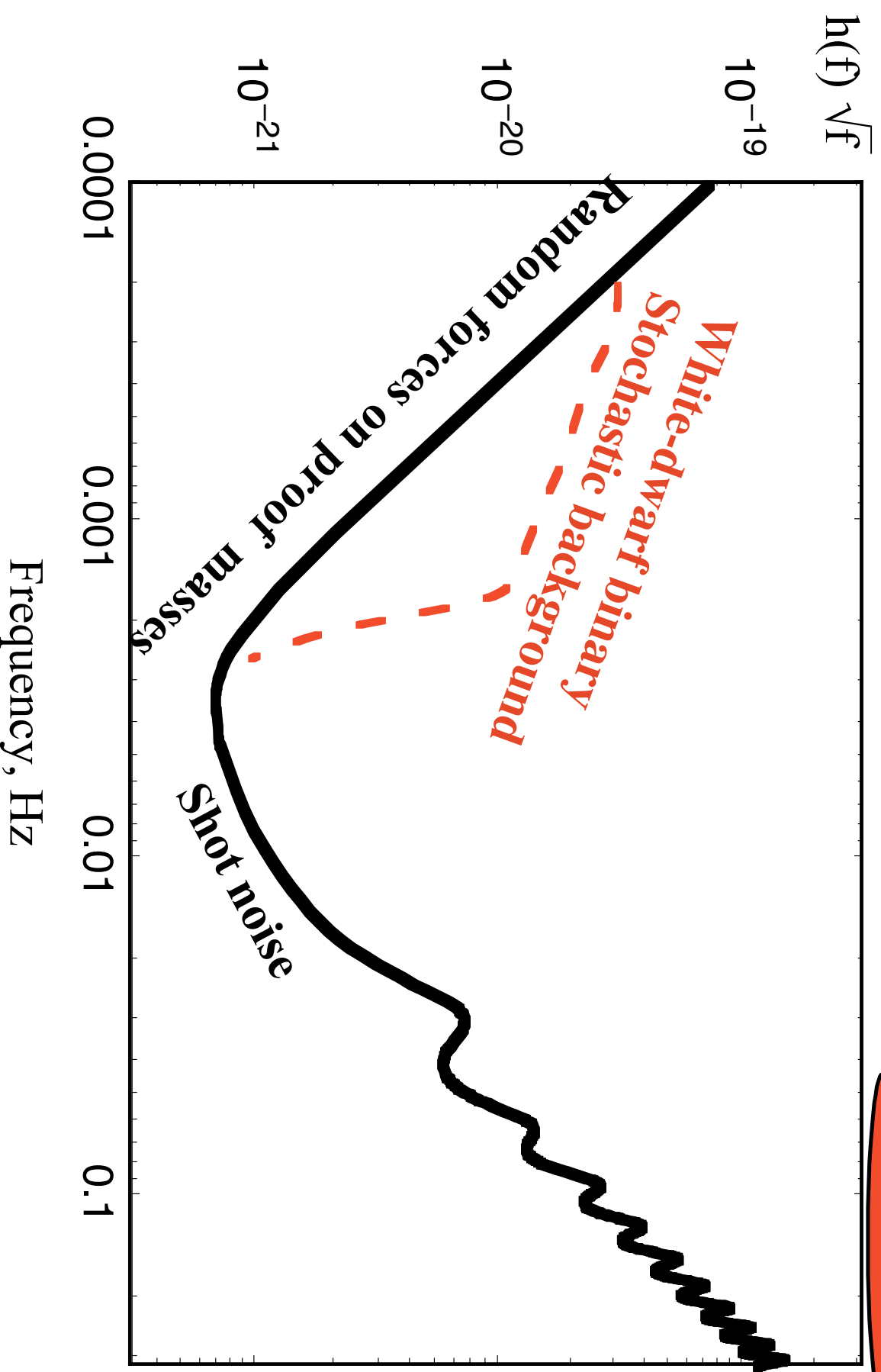
- Three “drag-free” spacecraft
- 5 million km separations
- 1 Watt laser, 30cm diameter telescopes
- Relative motions of spacecraft:
  - $\sim$  1 million wavelengths / sec
  - Light beams beat against each other (heterodyne detection); beat signal fourier analyzed
- **Joint American/European**
- **US: Managed at GSFC (Md)**
  - Payload & Science: JPL/Caltech
  - Tom Prince: Mission Scientist
- **Launch: 2011**

# LISA: The Technical Challenge



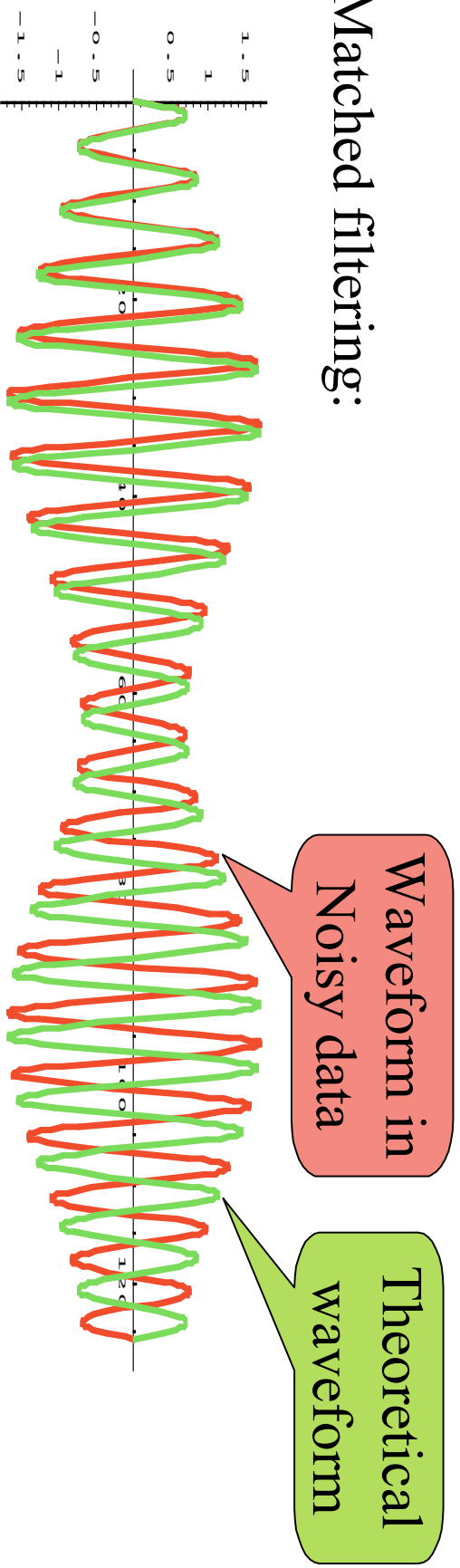
- Monitor the relative motion of the satellites’ “proof masses”, 5 million kilometers apart, to a precision
  - $\sim 10^{-9}$  cm [in frequency band  $f \sim 0.1 - 10^{-4}$  Hz ]
  - $\sim 10^{-5}$  of the wavelength of light
  - accelerations  $\sim 10^{-16}$  g
- Guarantee that the only forces acting on the proof masses at this level are gravitational, from outside the spacecraft

# LISA Noise Curve



# Gravitational-Wave Data Analysis

- Matched filtering:



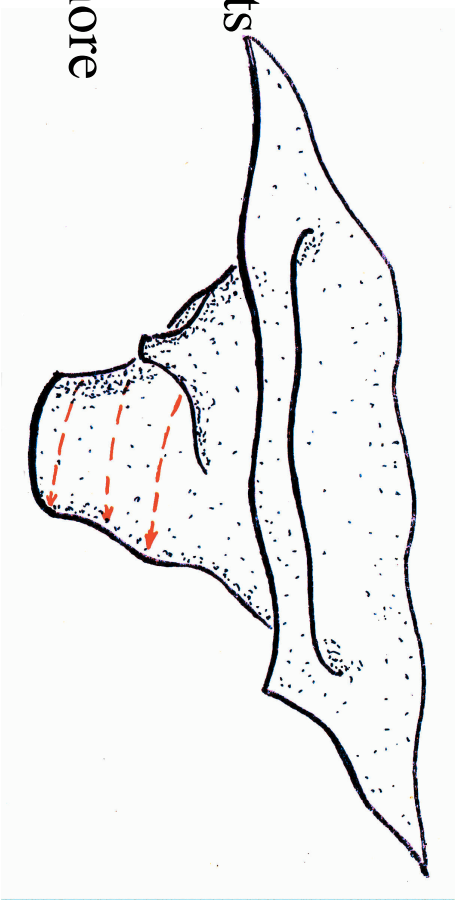
- If waveforms slip by  $\sim 1$  radian, it is obvious in cross correlation
- LIGO: up to  $\sim 20,000$  cycles ( $\sim 100,000$  radians)
- LISA: up to  $\sim 200,000$  cycles ( $\sim 1$  million radians)
- Theoretical challenge: compute waveforms to this accuracy
- If waveforms poorly known:
  - Must use other analysis methods: significant loss of signal strength!
    - e.g. Flanagan's excess power method: filter  $h(t)$  then square & integrate.

# Scientific Goals of LIGO and LISA

- **Astronomy: Open up a Radically New Window Onto the Universe**
- **Physics: Convert the study of highly curved spacetime**
  - From a purely theoretical enterprise (exploring general relativity theory)
  - To a joint observational/theoretical enterprise
- **Examples: Sources organized by science we expect to extract, not by when they might be detected --**

# The Inspiral of a White Dwarf (WD), Neutron Star (NS), or Small Black Hole (BH) into a Supermassive BH

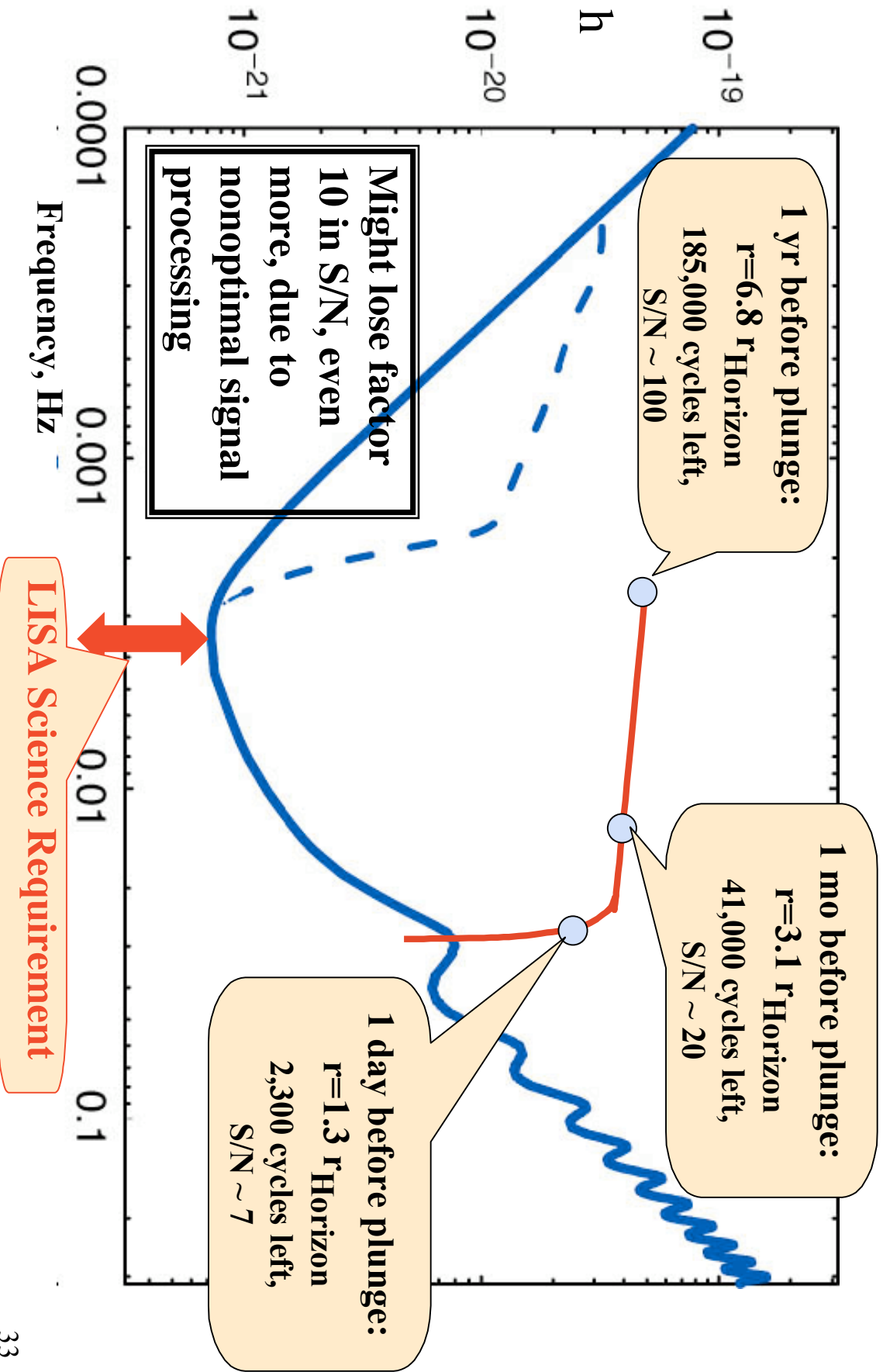
- **Astrophysical phenomenology:**
  - Occurs in nuclei of galaxies
  - Provides a probe of the environments of supermassive holes
  - Rates: a few per year; perhaps far more
- **Frequency band and detectors:**
  - Low frequencies; LISA
- **Information carried by the waves:**
  - High-precision map of the spacetime curvature of the supermassive BH



- **Science to be done:**
  - Map black holes, test “no hair theorem”, test theory of evolution of black-hole horizons when gravitationally perturbed, observe extraction of spin energy from black holes.
- **Method of computing waveforms:**
  - Black-hole perturbation theory; radiation-reaction theory

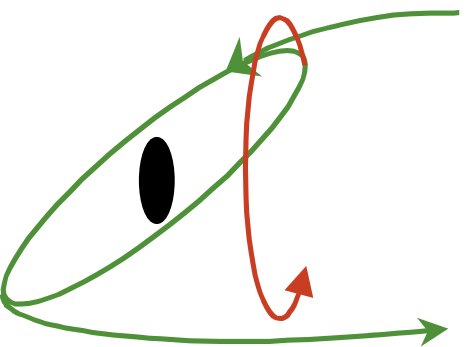


**LISA Inspirar Example: Circular, Equatorial orbit;  
 10 Msun /  $10^6$  Msun; fast spin -- @1 Gpc [optimistic]  
 (pessimistic: signal 10 times weaker)**



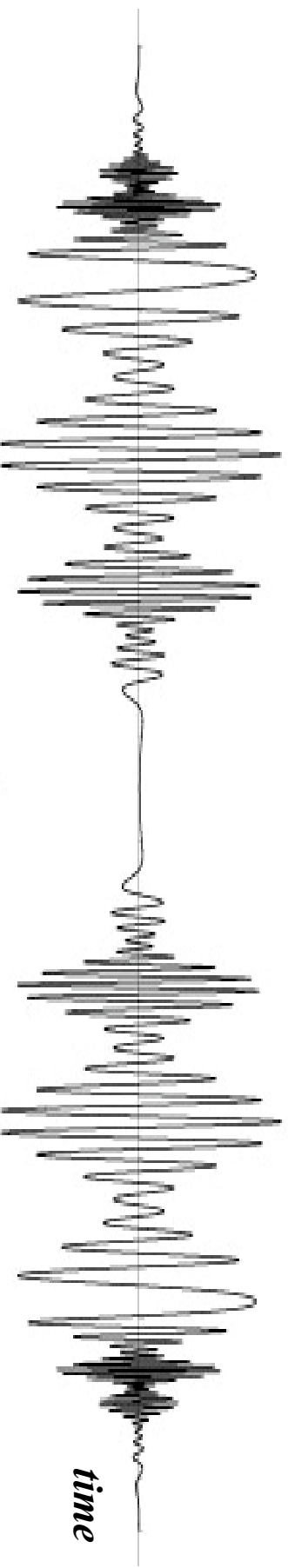
# Inspirational Waves: Why might signal processing be non-optimal?

- Typical Orbit in last year:

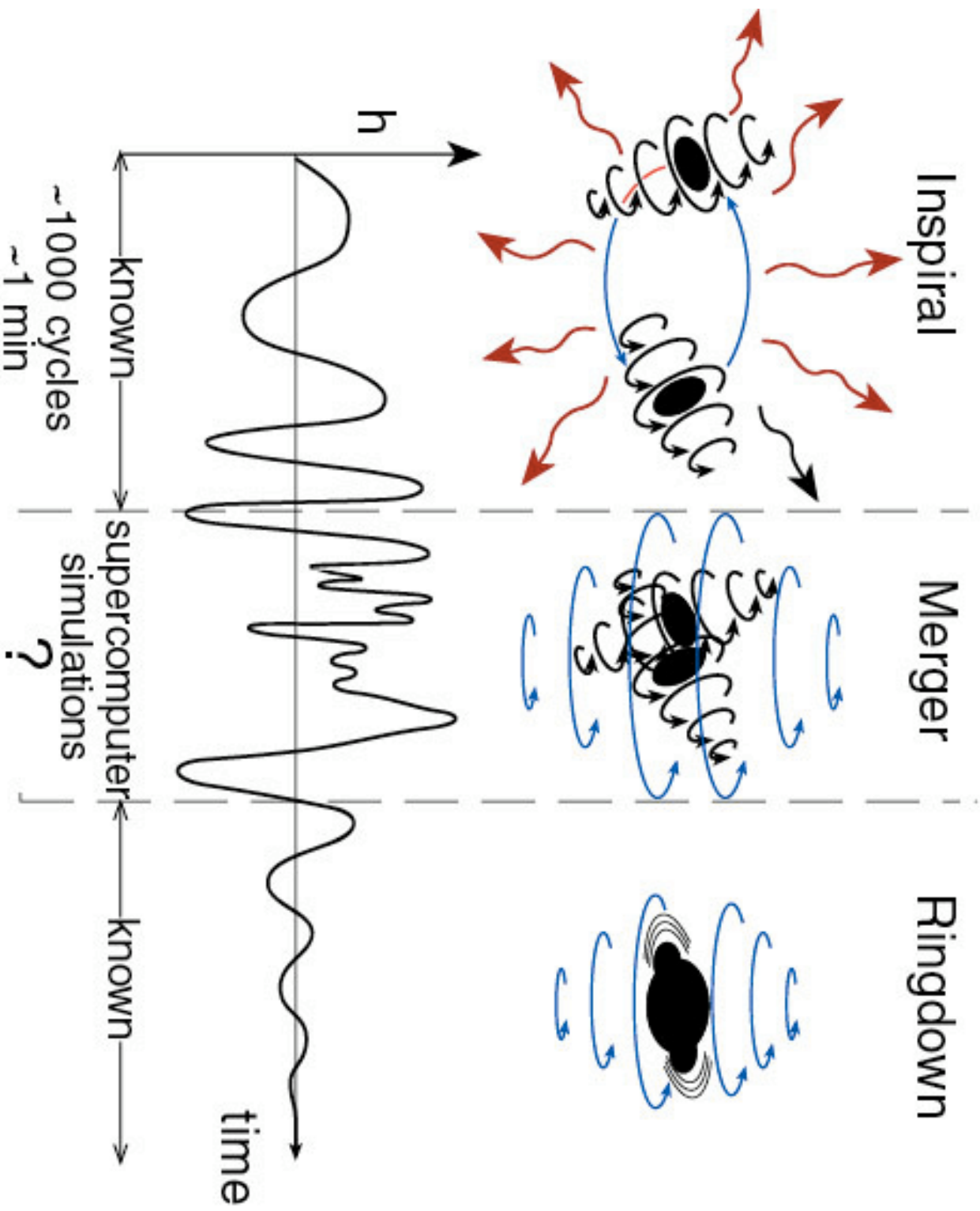


- Extreme sensitivity of orbit to initial conditions => ?? Coherent matched filtering no longer than a few days ?? Less?
- Many distant inspirals may give troublesome stochastic background; hard to separate strongest inspirals
- *To explore & quantify this: need waveforms. Will take ~2 years of concerted effort to produce them & quantify loss of S/N*

- Corresponding Waveform [schematic]:

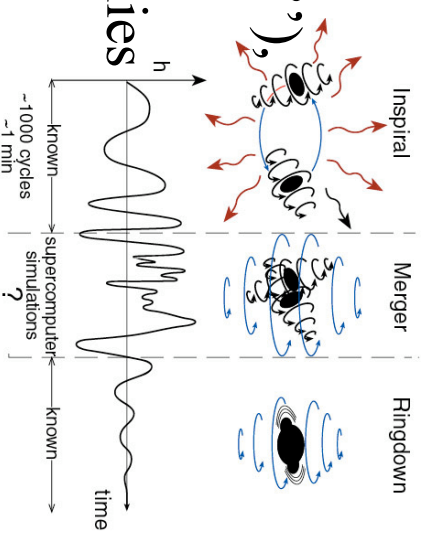


# Binary Black Hole Mergers



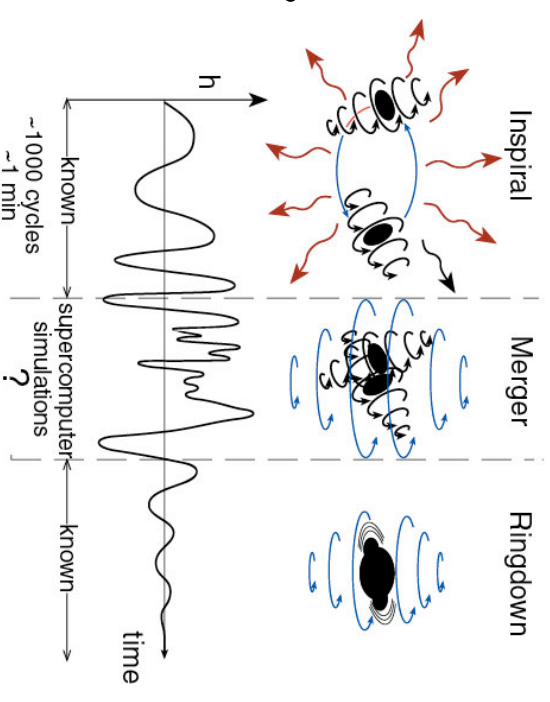
# Binary Black Hole Mergers [cont.]

- **Astrophysical phenomenology:**
  - Stellar-mass holes: in bodies of galaxies (‘‘field’’), in globular & other clusters.
  - Supermassive holes: as result of merger of galaxies
- **Frequency band and detectors:**
  - Stellar-mass: High frequencies; LIGO & partners
  - Supermassive: Low frequencies; LISA
- **Rates, Signal to noise ratios:**
  - LIGO, initial interferometers: seen to 100Mpc,  $\sim 1/200\text{yr}$  to  $\sim 1/\text{yr}$ ; S/N  $\sim 10$  or less
  - LIGO, advanced interferometers: seen to  $z\sim 0.4$ ,  $\sim 2/\text{mo}$  to  $\sim 15/\text{day}$ ; S/N  $\sim 10$  to 100
  - LISA: seen to  $z\sim 10s$  (earliest objects in universe),  $\sim \text{few}/\text{yr}$ ; S/N  $\sim 100$  to 100,000



# Binary Black Hole Mergers [cont.]

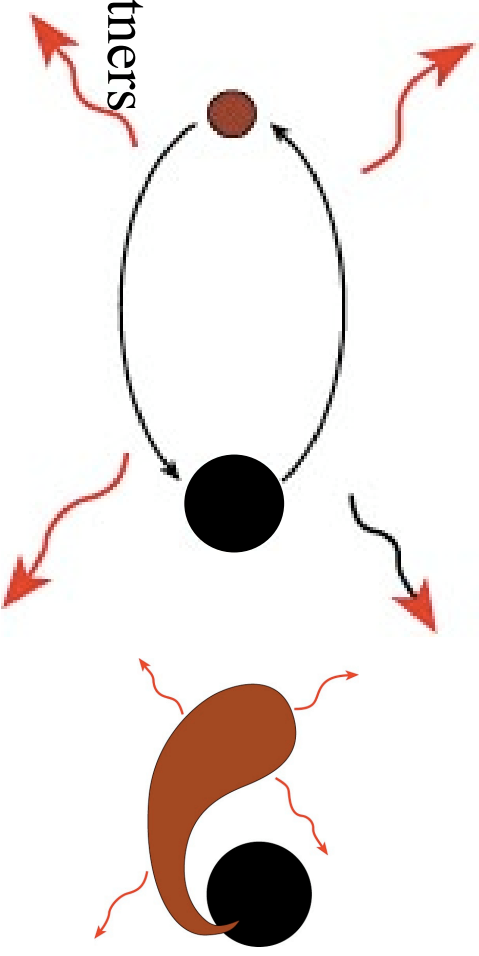
- **Information carried by the waves:**
  - Inspiral: Masses, spins, surface areas, and orbits of initial holes
  - Merger: The highly nonlinear dynamics of curved spacetime
  - Ringdown: Mass, spin, surface area, ...



- **Science to be done:**
  - Test Penrose’s cosmic censorship conjecture
  - Test Hawking’s second law of black hole mechanics (horizon area increase)
  - Watch a newborn black hole pulsate, radiating away its excess “hair”
  - Probe the nonlinear dynamics of spacetime curvature under the most extreme of circumstances that occurs in the modern universe
  - Probe demography of black hole binaries
- **Methods of computing waveforms:**
  - Inspiral: post-Newtonian expansion; merger: numerical relativity; ringdown: black-hole perturbation theory

# Neutron-Star / Black-Hole Mergers

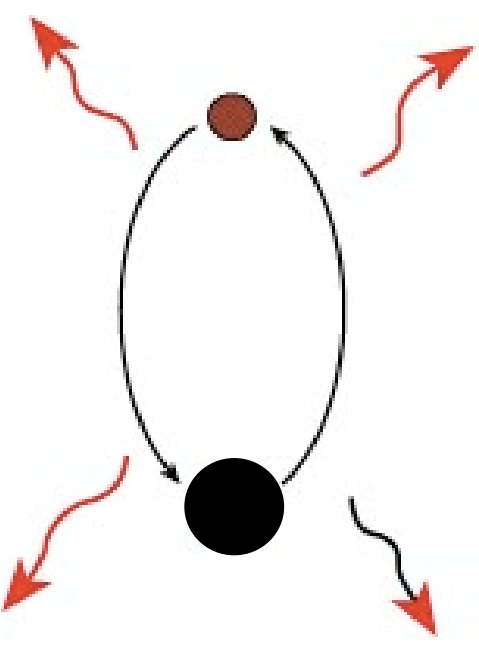
- **Astrophysical phenomenology:**
  - Stellar-mass objects: in field, in globular & other clusters.
- **Frequency band and detectors:**
  - High frequencies: LIGO and partners
- **Rates:**
  - Initial IFOs: 43Mpc, 1/2500yrs to 1/2yrs
  - Advanced IFOs: 650Mpc, 1/yr to 4/day



- **Information carried by waves:**
  - Inspiral: masses, spins, orbit
  - Tidal disruption of NS: neutron-star structure (e.g. radius)
- **Science to be done:**
  - Probe neutron-star structure, equation of state of matter
- **Methods of analysis:**
  - Inspiral: post-Newtonian; disruption of NS: numerical relativity

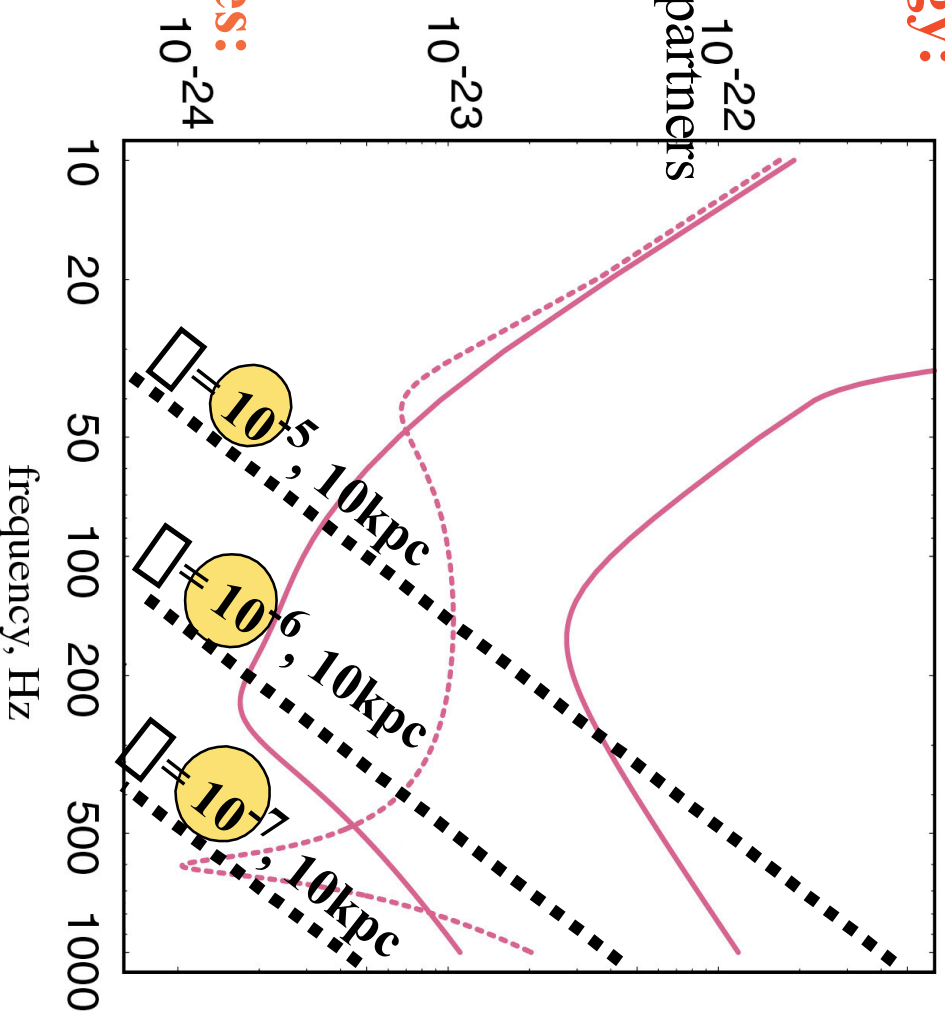
# Neutron-Star / Neutron-Star Inspiral

- **Astrophysical phenomenology:**
  - Main-sequence progenitors in field, capture binaries in globular clusters
- **Frequency band and detectors:**
  - High frequencies: LIGO and partners
- **Rates:**
  - Initial IFOs: 20Mpc, 1/3000yrs to 1/3yrs
  - Advanced IFOs: 300Mpc, 1/yr to 3/day
- **Information carried by waves:**
  - Inspiral: masses, spins, orbit
  - Merger: probably lost in LIGO's high-frequency noise
- **Science to be done:**
  - Test relativistic effects in inspiral [also for NS/BH and BH/BH]
- **Methods of analysis:**
  - Post-Newtonian expansions



# Spinning Neutron Stars: Pulsars

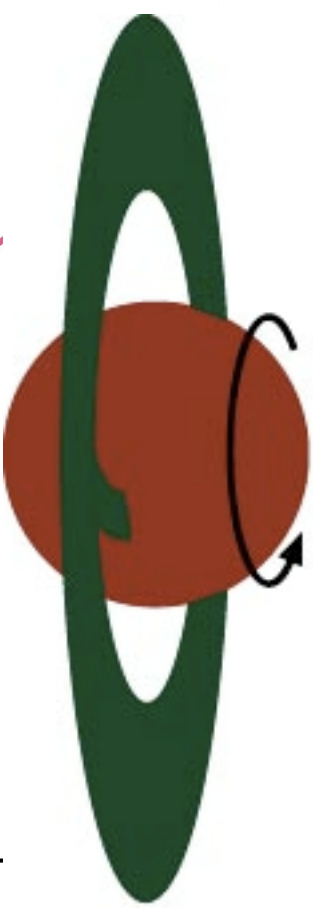
- **Astrophysical phenomenology:**
  - Pulsars in our galaxy
- **Frequency band and detectors:**
  - High frequencies: LIGO and partners
- **Detectability:**
  - Governed by ellipticity, spin
  - Ellipticities thought to be  $\lesssim 10^{-6}$ ; possibly  $10^{-5}$
- **Information carried by waves:**
  - NS structure
  - Behavior in quakes
- **Methods of analysis:**
  - Slow-motion, strong-gravity



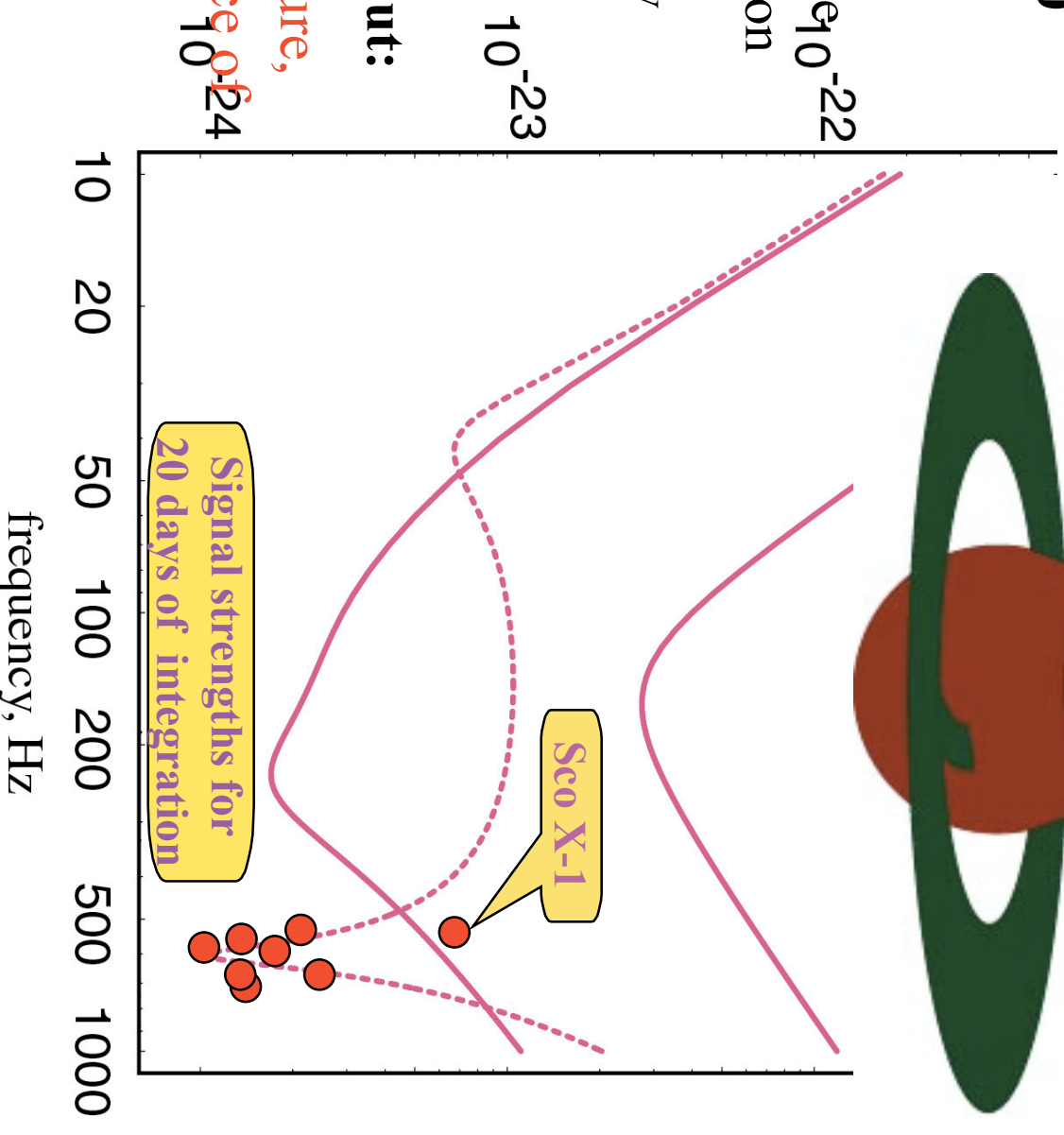


# Spinning Neutron Stars:

## Low-Mass X-Ray Binaries in Our Galaxy [LIGO]



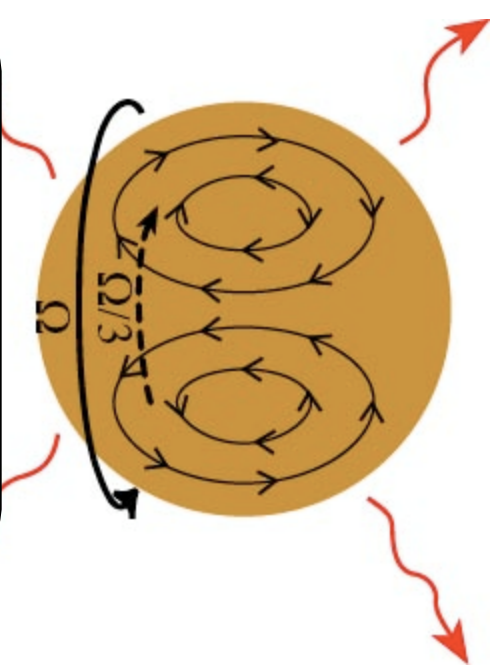
- Rotation rates  $\sim 250$  to  $700$  revolutions / sec
  - Why not faster?
  - **Bildsten**: Spin-up torque  $10^{-22}$  balanced by GW emission torque
- If so, and steady state: X-ray luminosity  $\sim$  GW strength
- Combined GW & EM obs's  $\Rightarrow$  information about:
  - crust strength & structure,
  - temperature dependence of  $\nu$  viscosity, ...



# Neutron-Star Births:

## R-Mode Sloshing in First ~1yr of Life [LIGO]

- NS formed in supernova or accretion-induced collapse of a white dwarf.
  - If NS born with  $P_{\text{spin}} < 10$  msec:
    - Gravitational radiation reaction drives sloshing
- Physics complexities:
  - What stops the growth of sloshing & at what amplitude?
  - Crust formation in presence of sloshing?
  - Coupling of R-modes to other modes?
  - Wave breaking & shock formation?
  - Magnetic-field torques?

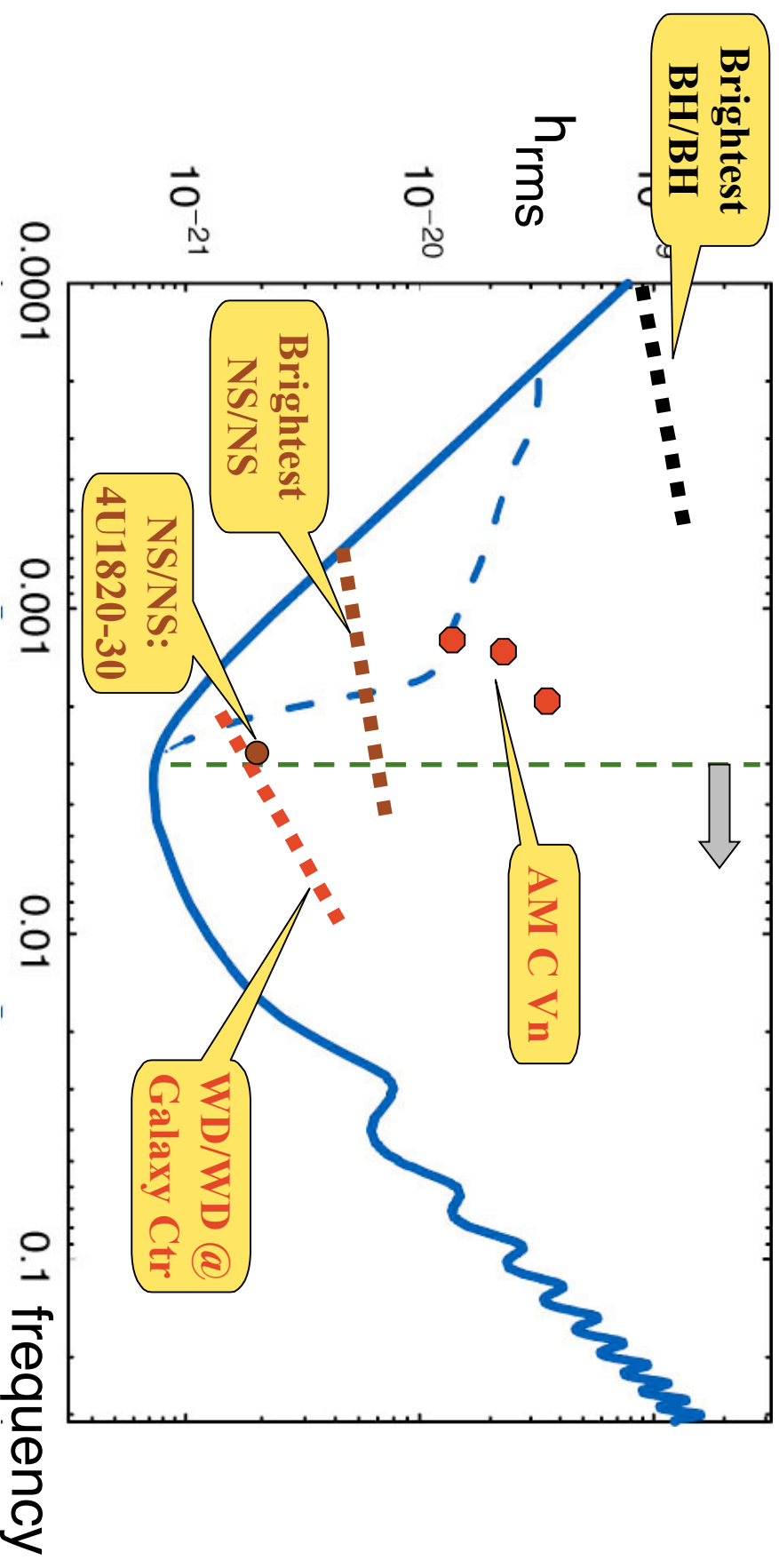


Depending on this, GW's may be detectable out to Virgo (supernova rate several per year). BUT recent research pessimistic

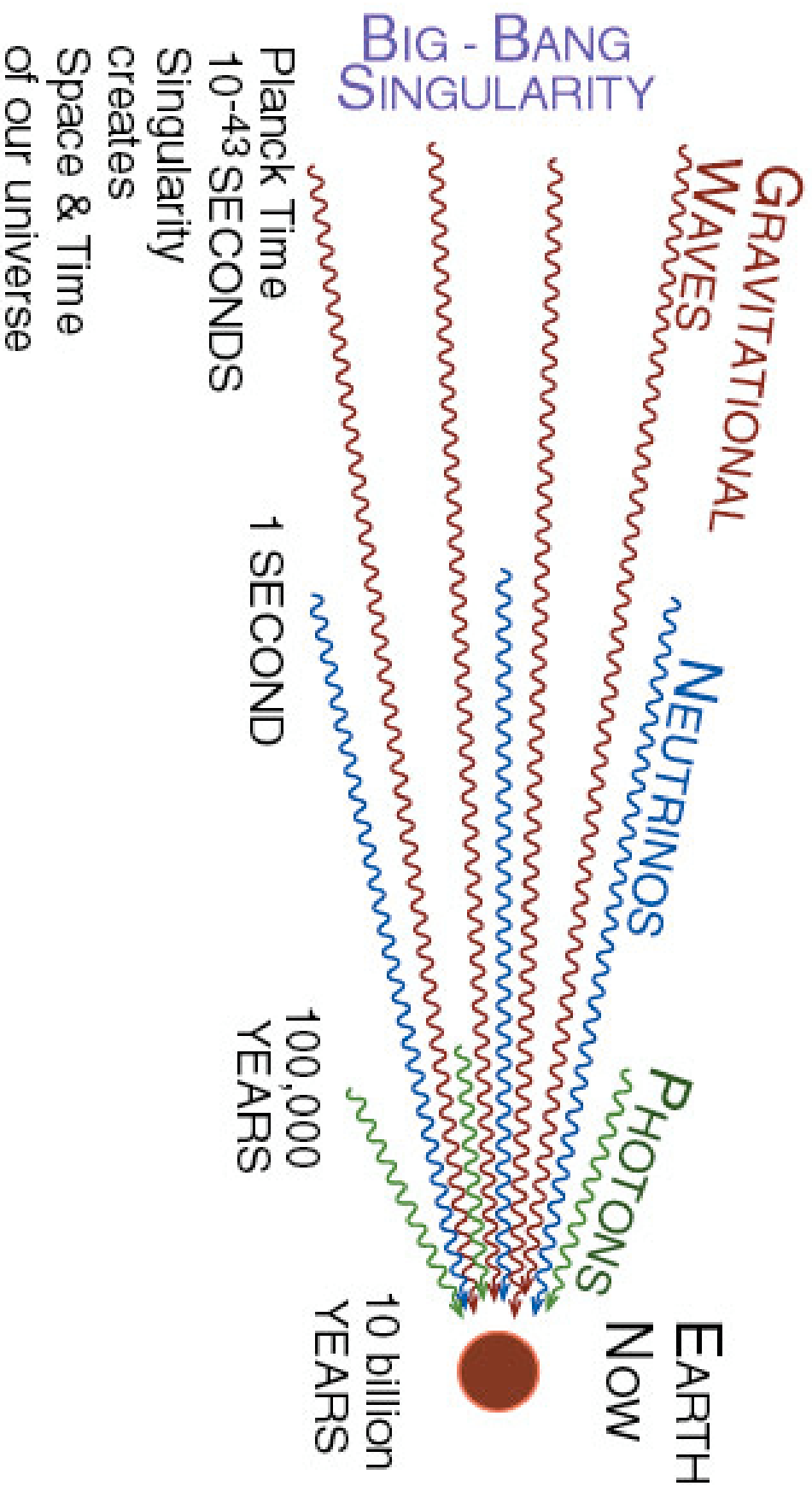
GW's carry information about these

# COMPACT BINARIES IN OUR GALAXY: LISA

- Censuses of short-period compact binaries in our Galaxy; rich astro
- BH/BH studies: e.g. merger rate; compare with LIGO
- NS/NS studies -- e.g. merger rate; compare with LIGO et al
- 3000 WD/WD binaries will stick up above the WD/WD noise
- Inspirals (&  $M_{\text{chirp}}$ ) will be measured if  $f > 0.003$  Hz

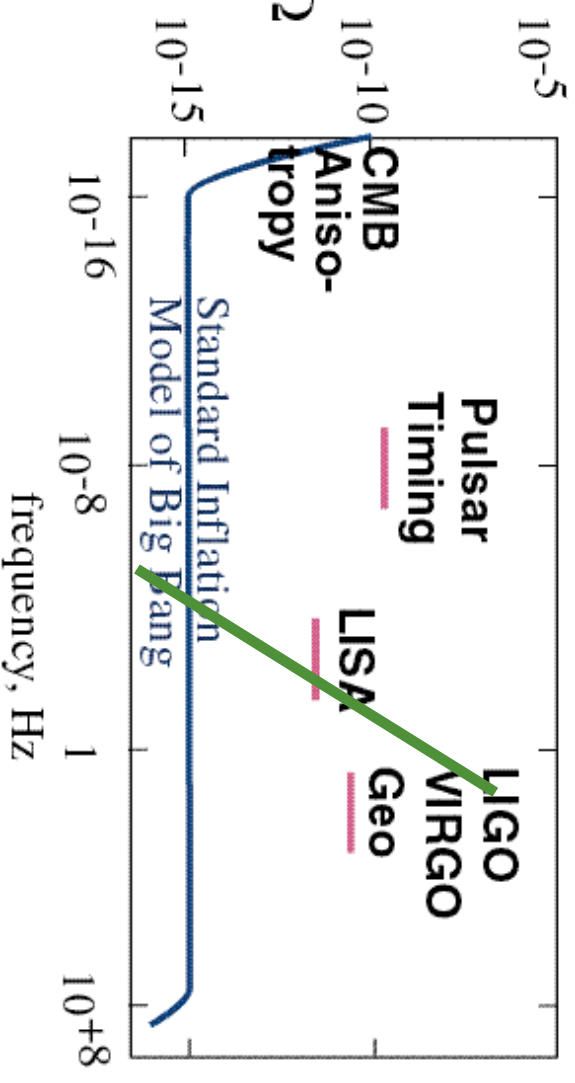


# The First One Second of Universe's Life



# Waves from Planck Era, Amplified by Inflation

- **Cosmological phenomenology:**
  - Vacuum fluctuations (at least) created in Planck era
  - Amplified by interaction with background spacetime curvature of universe during inflation
- **Frequency band and detector**
  - All bands, all detectors
- **Strength predictions:**
  - “Standard Inflation”: detect  $\Omega$  only in ELF band (CMB)
  - “Pre-big-bang”, etc: more 0
- **Information carried:**
  - Physics of big bang, inflation; equation of state of very early universe
- **Methods of analysis:**
  - Cosmological perturbation theory; quantum gravity



# Phase Transitions in Very Early Universe

- **Cosmological Phenomenology:**
  - As universe expanded, fundamental forces decoupled from each other; phase transition at each decoupling produced gravitational waves; GW's redshifted with expansion
- **Frequency bands and detectors:**
  - LISA probes Electroweak Phase Transition ( $\sim 100$  GeV) at universe age  $\sim 10^{-15}$  sec
  - LIGO probes any phase transition that might have occurred at  $\sim 10^9$  GeV and age  $\sim 10^{-25}$  sec
- **Science:**
  - Probe high-energy physics, e.g. strength of electroweak phase transition; probe topological defects & evolution of inhomogeneities produced by phase transition

# Mesoscopic Oscillations in Very Early Universe

- Recent *speculations* about our observed universe as a 3-dimensional defect (*brane*) in a higher dimensional universe:
- All fundamental forces except gravity are confined to the brane.
  - Gravity is confined to some distance  $b < 1$  mm from the brane, in the higher dimensions, and feels the shape of the brane.
- Excitations of our brane: [Craig Hogan]
  - Brane forms wrinkled on all scales up to  $b$ . Wrinkles evolve dynamically, producing GW's, with energy densities  $\sim$  those in other forms of radiation.
  - GW's from excitation scales  $\sim 10$  A to 1 mm get redshifted to LISA band with GW strengths easily detected by LISA.
  - GW's from scales  $\sim 10^{-10}$  to  $10^{-13}$  mm redshifted to LIGO band.

